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HOW THE EARTH IS MEASURED.*

BY PROF. J. HOWARD GORE.

[*A lecture delivered before the Franklin Institute, November 30, 1891.*]

[*Concluded from p. 366.*]

The name which will always be associated with the early history of Geodesy is that of Willebrord Snell (or Snellius), a Netherland geometer. He has rendered the first years of the seventeenth century luminous by the brilliancy of his mathematical conceptions and the boldness with which he followed up his hypotheses. To him belongs the credit of having first promulgated the theory of the refraction of light and the discovery of the three-point problem, usually ascribed to Pothenot, while he enriched the literature of astronomy by the publication of an important paper on comets.

* Certain portions of this paper are taken from the author's work on *Geodesy*, published by Houghton, Mifflin & Co.

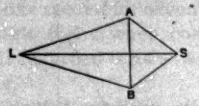
The year 1615 carries with itself the glory of having witnessed the inception of that method which will always be used in geodetic determinations—that is, the system of triangulating from a known base, in this way reducing the probability of error in its greatest source—the measurement of a long line.

As this general plan has been followed by all who have subsequently made attempts to determine the length of a terrestrial arc, the method of Snell deserves a complete description.

It is said that Ptolemy pointed out the fact that in order to determine the length of an arc it was not necessary to measure along a meridian. Still no one was willing to accept this belief until Snell demonstrated it to be true. He went still further, saying that since it is not essential to measure on a meridian, it is not absolutely necessary that the terminal points should be connected by a straight line, but the line may be broken. This, perhaps, suggested at once that at least some of the lines might have their lengths computed, thereby saving the trouble of measuring them. It was, of course, known at that time that in a triangle, if one side and the angles be given the remaining sides can be found. The known side might be short, while the computed sides could be relatively much longer. From this it was but a step to realize that a side which has been computed in one triangle may become the known side in an adjoining triangle and aid in determining the remaining sides of the latter. Thus, triangle could be joined to triangle link by link—forming what is now called a chain or net—with only one side, called the *base*, determined by direct measurement.

His operations stretched over the arc between Alcmaar and Bergen op Zoom, and was computed in the following manner: He measured with a chain a base that was perpendicular to the line joining Leyden and Soerterwoude, *A B* in the appended figure; this was 326·4 Rhenish rods. With this line known and all the angles of the triangles *L A B*, *A B S*, and *L A S*, he computed *L A* as a side of the first-named triangle and *A S* a side of the second; then

with *L A*, *A S*, and the angle *L A S* known, the side *L S* was computed, giving this as a new base. Upon this he formed a new triangle, and continued so doing until he had

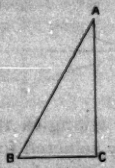


a chain of thirty-three triangles reaching a church spire which he had previously selected for the terminal point.

The angles were measured with a graduated semi-circle of three and one-half feet diameter. This size was necessary as it was not until sixteen years later that the vernier was introduced to aid in reading angles. This semi-circle was likewise unprovided with telescopes, so that the observer had to rely on the unaided eye in sighting to distant objects. Snell realized that this crude method was liable to introduce uncertainties, so he sought to reduce them to a minimum by measuring all the angles of each triangle and computing each side through different routes. As these variously obtained values were in all cases quite harmonious, he naturally congratulated himself upon the attainment of correct results.

Now, knowing the length of each side, the summation of a set of contiguous lines gave the length of the *broken* line joining his terminal points. However, these lines not having the same direction it was necessary to calculate what the length of each line would be if it had the direction of the line joining the ends of his arc; that is, he found the *projection* of each line upon this direction; then the sum of these projections gave the distance required, which he found to be 34710·06 rods. With this value for *A B* it was necessary to find *A C*, the meridian distance between *A*, Alcmaar, and *C* the parallel of *B*, Bergen op Zoom. Here again the question was simply to find the projection of *A B* on a north and south line. For this he needed to know in addition to *A B*, the angle *B A C*, or the *azimuth* of the oblique arc *A B*. For the purpose of determining this azimuth, Snell carefully fixed from astronomic observations

the meridian of Leyden, and then measured at Leyden, the angle which one of the sides in his chain made with this meridian. By combining this azimuth, or angle of position



as he called it, with certain angles in the chain, he obtained the azimuth of *A B* or the angle *B A C*. The result of this computation gave 33,930 rods for *A C*.

With the length of the arc known, nothing remained but to know its amplitude, or the difference in the latitude of Alcmaar and Bergen op Zoom. The latitudes were determined from observations made with a quadrant of five and one-half feet diameter, likewise without a telescope. The results of his observations gave:

Latitude Alcmaar, ...	52°	40′	30″
Bergen, ...	51°	39′	00″
Amplitude of the arc, ...	1°	11′	30″

So we have for a degree $\frac{60}{71.5}$.33930 or 28,473 rods. By taking the arc from Leyden to Alcmaar he found for a degree 28,510 rods; from these two discordant results he arbitrarily assumes 28,500 rods for a degree.

As most of the measures of this period are given in toises (a toise being approximately 6·4 feet), the length of a degree as given by Snell is 55,072 toises, counting the rod at 1·9324 toises.

Since the error in this length of a degree is practically 2,000 toises, while the method is, in general terms, correct, it is well to look into the sources of this discrepancy. The first which attracts our attention is the fact that he regarded the surface of the earth as a plane, and computed all of his triangles by plane instead of spherical trigonometry, without making any correction for *spherical excess*—the amount

by which the three angles of a spherical triangle exceed 180°. This negligence, however, would introduce no serious error, as none of the triangles were large, and the spherical excess is only about one second for seventy-five square miles. But in the last triangle *B A C* when computing the length of the *arc A C*, Snell regarded it as a *chord*, thereby making an error of one toise.

Had his vertices or stations been at different elevations it would be necessary to reduce the angle which he observed between two stations of different elevations to its equivalent in the plane of either, because his instrument was not level, but so canted as to be in the oblique plane of the two stations—so that the angle read was oblique and not horizontal. The modern method of observing with the circle level provided with a telescope having a vertical motion enables one to ascertain at once the horizontal angle between points of different elevations. Then we project all vertices upon the surface of an ellipsoid which differs the least possible from the mathematical shape of the earth and bind these points with lines upon this surface. After deducting equally from each angle the computed spherical excess, the computations are carried forward in accordance with the established principles of plane trigonometry.

Thus far the sources of error pointed out in this pioneer work are insignificant. Those that remain are: base-measuring, angle-reading and astronomic determinations.

Snell himself suspected the existence of the two first named and went so far as to re-measure his base, which he did with wooden rods on the ice which was formed on the overflowing waters around Leyden in 1622. A triple measurement of this base satisfied him that his linear determination was good. He then repeated the measurement of the angles and found some discrepancies which caused him to say in his note-book that his last readings were the only reliable ones. By this time his advanced age prevented him from recomputing all the triangles—a labor at that time very onerous, owing to the non-existence of logarithmic tables.

About 100 years later Musschenbroek, a descendant of

Snell, revised the entire work, using a quadrant with a telescope attached. With this instrument he detected a number of errors in the angles of his predecessor, which, with a new determination of the latitudes of the terminal points, gave a value very nearly correct, according to our present judgment.

Modern practice differs from Snell's operations in character rather than kind. We measure now a short line three or four miles in length, erect upon it sides of a triangle, add triangle to triangle, forming a chain, project the continuous broken line upon a meridian, determine the amplitude of the arc by obtaining the latitudes of its termini and thus find the length of a degree.

But in measuring the base we exercise the greatest caution, in obtaining the exact length of the measuring unit in terms of a standard, and in securing conditions such as insure exact repetitions of this unit. So that when we say the base contains the unit a certain number of times, we know that the length will be expressed by that number of times a known quantity. Those who have had experience with instruments of precision know how difficult it is to obtain the length of one unit in terms of another; and all who have employed standards of measure, or sought constant temperature will realize how hard it is to repeat operations under similar conditions. Notwithstanding these difficulties great accuracy is possible. Last summer it was my good fortune to participate in the measuring of a base. We obtained two results for its length, differing from one another by a quantity less than one-fifth of an inch—a discrepancy remarkably small when we consider that the length of this base was more than three miles.

In the matter of triangulation improved angle-reading instruments are now employed, and extra precautions are taken to observe under such conditions as to eliminate errors of refraction, or errors resulting from faulty construction of the instrument. That this has been successfully accomplished can be seen from the statement that the measured length of the Olney base differed from the value obtained through triangulation from the Chicago base—200

miles distant—by *two* inches—and this achievement does not stand alone.

The astronomic determinations—one of the most important features—have likewise improved in accuracy, and calling electricity to their aid, astronomers are now enabled to ascertain with requisite precision the amplitude of an arc of parallel by determining the difference of longitude of the terminal points.

In this advance there was demonstrated beyond doubt the fact that a degree of the equator had not the same length that a degree of meridian had. This had been suspected long before astronomy gave the casting vote, and as if to complicate the problem degrees of the same meridian seemed to have lengths varying with the latitude. Theory and pure mathematics in the wizard hands of Newton made ellipses of the earth's meridians—ellipses whose shorter axis was the axis of rotation. At the poles, therefore, the curvature, the amount of change or deflection from a straight line was less than at the equator, and as a greater circumference and hence longer degree go with the less curvature, a degree near the pole would be longer than towards the equator.

Inseparably connected with the name of Newton is that of Huygens. His attention was first drawn to the subject of the figure of the earth by the variation in the length of the seconds pendulum in different latitudes, which was first announced as an observed fact when Richer returned in 1672 from Cayenne. It was quite natural that Huygens should take notice of everything related to pendulum behavior, as he was at this time busy with the application of the pendulum to the regulating of clocks. He immediately perceived that this phenomenon—the shortening of the seconds pendulum on approaching the equator—was caused by the centrifugal force at the earth's surface, which, increasing as the equator is approached, lessens the power of gravity and retards the time of the pendulum's vibration. It also occurred to him that if the earth were a perfect sphere, a plumb-line would not be at right angles to the sea, or to the surface of standing water, but would be

deflected somewhat by the action of centrifugal force. Hence a light body in still water would not press perpendicularly upon the surface, and consequently could not be at rest, which is contrary to experience. Huygens, therefore, argued that the earth was not spherical, but protuberant at the equator, so that the terrestrial meridians might everywhere be perpendicular to the plumb-line. Here he ceased speculating, perhaps waiting for observations to in some way substantiate his views, nor did he proceed until Newton furnished the stimulus in the nature of a suggestion.

Newton determined the ratio of the two axes by conceiving two columns of fluid to extend from the centre of the earth outward towards the surface; one to the equator, the other to one of the poles. Since these two columns were in equilibrium they would press upon one another with equal intensity, so that the ratio of their lengths would be found by comparing their weights. The weight of the equatorial column is equal to gravitation diminished by centrifugal force, while the polar column, unaffected by the earth's diurnal motion, had a weight dependent solely on the gravitation of its particles.

This centrifugal force of each equatorial particle depended upon its angular velocity and its distance from the centre, but its gravitation, resulting from the combined attraction of the surrounding particles, was one of the problems first solved by Newton. The result of this discussion gave as the ratio of the polar to the equatorial axis, 229 to 230. This is usually stated $b : a :: 229 : 230$, and the ellipticity, which is the ratio of $a - b$ to a is $\frac{1}{230}$.

This was the needful suggestion for Huygens. He at once took up the problem, but rejecting the Newtonian principle of an attraction between the particles, he placed in the centre of the mass a force attracting the particles according to the inverse square of the distances. Upon this hypothesis he proved that a homogeneous body of fluid revolving upon an axis will be in equilibrium when it has the figure of an oblate spheroid very little different from a sphere, the ellipticity being one-half of the ratio of centrifugal force to the gravity at the equator, or $\frac{1}{2} \times \frac{1}{289} = \frac{1}{578}$.

He tried to verify his theory from actual experiment, so he took a soft globe of clay and attached it to an axis. This he caused to revolve very rapidly, when he observed that the ball became flattened at each end of the axis and enlarged at the middle.

He very cleverly concluded from this theory that the water adjusts itself over the surface of the earth and that the fixed land must do the same—for if the land at the poles were exactly at the same distance from the centre of the earth that the equator is, the water of the sea in this latter region would be raised above the land. But since we find at the equator large areas of land, these too must have been thrown out under the action of centrifugal force, and for this to be possible, the earth was at one time in a fluid condition, becoming solid later in its life. Quite recently Professor Stokes has enunciated almost the same idea when he said that the fact that the waters of the earth are in equilibrium shows that at some time there must have been a bulging out of the land in the equatorial regions. Newton saw in the flattening of Jupiter as reported by Cassini (1691) a proof by analogy of the oblate hypothesis.

The acceptance of a flattened earth was by no means instantaneous. Terrestrial measures were not harmonious, but their weight of evidence was on the other side or towards an elongated earth. This caused a conflict—notable in the history of Geodesy sending champions of the latter hypothesis to Peru and within the polar circle to bring back as the fruits of years of toil the lengths of an equatorial degree and of a polar degree. The evidence was on the unexpected side—the earth was flattened. However, the adherents did not at once relinquish the elongated theory, but computed and re-computed the tell-tale figures, discussed and re-discussed the methods which yielded such unwelcome results.

Knowing that the earth is round like an orange and flattened at each pole is not sufficient, the amount of this flattening is of great importance—it enters as a potent factor into all computations of directions and distances which extend over any considerable portion of the earth's surface.

With an erroneous value .or this flattening—this ellipticity —maps would be a contorted representation of countries, boundary lines would cause endless confusion and ships often follow a plotted course to their destruction.

The earth's diameter is often the astronomer's unit, and furnishes him with a base on which to construct triangles to bring the starry bodies under our grasping measures. A faulty value for this unit retarded for twenty years the discovery of the theory of universal gravitation, and with its correction that principle—finite in demonstration was applied to the infinite and worlds were harnessed with invisible traces.

To know the length of degree, to ascertain the shape of the earth, men have braved the cold within the Arctic circle, endured the heat of the equatorial regions, and penetrated India's malarial jungles. Peaks have been climbed, deserts traversed, and hostile tribes subjugated. To the theoretical side scores of the world's most profound mathematicians have devoted their time, contributing 6,000 treatises to the literature of the subject, while the practical side has been pushed ahead by the energies of countless troops of observers, artisans and laborers, supported by the expenditure of millions upon millions of dollars, and is advanced at this time by the united efforts of sixteen nations, each working within its own territory, but with a common plan and harmonious methods.

Does our utilitarian friend listen patiently and say despondingly *cui bono?* One item of ignorance dispelled is like a grain of sand removed from the most delicate mechanism. One particle of knowledge diffused is a lubricant that may facilitate the movements in the vast cycle of sciences from centre to periphery.

The line joining two points of observation on the earth's surface is our base line for estimating the distance to the heavenly bodies. With an approximately correct value for this Newton demonstrated a principle as unlimited as the infinite. What a more accurate value may suggest is beyond conjecture.

He who makes a contribution to science benefits the

human race. He furnishes a stimulus whose action is above conception, and plants a seed whose fruit may nourish coming ages.

Morse invented the telegraph but had to wait for Henry's induction coil. The telephone was a useless toy in Germany until Bell made electricity the vibrating agent. Adams saw Neptune in the perturbations of Uranus before it was visible to Galle's cyclopian eye. Mendalaef pointed out by a Keplerian law three chemical elements when the catalogue was supposed complete.

Instances without number might be given showing how the world stood still awaiting a scientific discovery, how practice paused for theory, how the artisan's hand was freed by a savant's brain.

We do not glorify the Arctic explorer simply because he waved our flag over a parallel never before visited, but because of the direct visible result of the spirit of investigation, and the indirect encouragement for others to search more prolific fields.

The African traveller returning with the map extended over much that was hitherto unknown ground makes as his greatest contribution, the inspiration to observe what surrounds us, to widen our limits, and "to see books in running brooks."

The link binding animal to vegetable life may be verified by a fisherman finding a more simple example of sporadic existence. The earliest stage of life on this globe may be pushed down below the geologist's calendar by a child picking up a single fossil.

Let us then honor science for science sake as well as for its utilitarian results. Let us revere that motive which prompts men to measure the earth with a span or estimate the approach of a star by a spectroscopic line.

MAXIMUM STRESSES FROM MOVING SINGLE LOADS IN THE MEMBERS OF THREE-HINGED ARCHES.

By Emrick A. Werner.

[*Continued from p. 384.*]

MAXIMA.

C_{15} *maximum.* Maximum line $g_m = \frac{1,500}{65} = 23{\cdot}1$

Maximum span $= (g_m - a) + a = 23{\cdot}1$

$$\frac{dC}{dg}\,max = 20\,W_1 - 13\,W$$

Second driver of first engine in (15); fifth driver 2·25 from Abutment *A*.

To the right of the dividing line (15)

$$W = 140\ W_1 = 100$$

To the left of the dividing line (15)

$$W = 140\ W_1 = 75$$

whence

$$\frac{dC}{dg} = \begin{cases} 1,820 - 2,000 < 0 \\ 1,820 - 1,500 > 0 \end{cases}$$

and

$$R\,l = 19,248$$

$$R'\,l = 1,752$$

$$\text{Check } R\,l + R'\,l = 21,000 = 140 \times 150 = W\,l$$

$$R_{15} = 638$$

$$R_{75} = 8,748$$

$$H\,f = R\,\frac{l}{2} - R_{75} = 876$$

$$H\,y = H\,f \cdot \frac{y}{f} = 315$$

$$\text{Check } R'_{15} = 290$$

$$R'_{75} = 20$$

$$H'\,f = \frac{R'\,l}{2} - R'_{75} = 9,624$$

$$H\,f \div f = H = 29{,}200$$

$$C_{15}\ max. = \left(\frac{M}{h}\right)_{15} - H = 972 \div 22{\cdot}2 - H = 43{,}800 - 29{,}200 =$$

$$= + 14{,}600$$

$$\text{Check } \left(\frac{M}{h}\right)' - H = + 14{,}600$$

$$D_{\text{I}} = \left(\frac{M}{h}\right)_{15} - \left(\frac{M}{h}\right)_{0} = + 43{,}800$$

$C_{30}\ max.$—

Maximum span = 40·0

$$\frac{dC}{dg} = 4\ W_1 - 3\ W$$

Second driver in point (30), second tender wheel 5·75 from Abutment A.

$$H = 69{,}200 \left(\frac{M}{h}\right)_{15} = 1{,}702 \div 22{\cdot}2 = 123{,}400$$

$$C_{30}\ max. = \left(\frac{M}{h}\right)_{15} - H = + 54{,}200$$

$$D_{\text{II}} = \left(\frac{M}{h}\right)_{30} - \left(\frac{M}{h}\right)_{15} = + 67{,}700$$

$C\ \ max.$—

Maximum span = 53·0

$$\frac{dC}{dg} = 20\ W_1 - 17\ W$$

First driver on point (45), last wheel of first tender 6·0 from Abutment A, followed by 3·8 feet of 3,000 pounds per lineal foot.

$$C_{45}\ max. = \left(\frac{M}{h}\right)_{45} - H = 1{,}682 \div 78 - H = 108{,}300$$

$$D_{\text{III}} = \left(\frac{M}{h}\right)_{45} - \left(\frac{M}{h}\right)_{30} = + 79{,}300$$

$C_{60}\ max.$—

Maximum span = 63·2

$$\frac{dC}{dg} = 20\ W_1 - 19\ W$$

First wheel of first engine in (60), last wheel of tender 13·0 from Abutment *A*, followed by 10·8 feet of 3,000 pounds per lineal foot.

$$C_{60}\ max. = 976 \div 42 - H = 232{,}200 - 135{,}600 = 96{,}600$$

$$D_{IV} = \left(\frac{M}{h}\right)_{60} - \left(\frac{M}{h}\right)_{45} = +\ 200. -$$

Minima.—

C_{15} *min.*—

Minimum span = 126·9

$$\frac{dC}{dg} = 13\ W - 22\ W_3$$

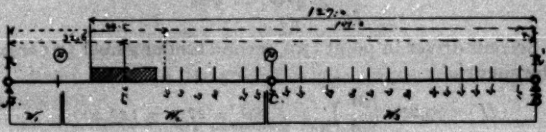


FIG. 17.

$$C_{15}\ min. = -\left[H + \left(\frac{M}{h}\right)_{15}\right] = -\ (364{,}400 + 37{,}100) =$$

$$= -\ 401{,}500 \text{ pounds.}$$

$$D_I = \left(\frac{M}{h}\right)_{15} - \left(\frac{M}{h}\right)_{0} = -\ 37{,}100 \text{ pounds.}$$

C_{30} *min.*—

Minimum span = 110·0

$$\frac{dC}{dg} = 15\ W - 22\ W_3$$

Third driver of second engine on top hinge.

$$C_{30}\ min. = -\ 443{,}700$$

$$D_{II} = -\ 58{,}200$$

C_{45} *min.*—

Minimum span = 97·0

$$\frac{dC}{dg} = 17\ W - 22\ W_3$$

Fourth driver of first engine on top hinge.

$$C_{45}\ min. = -\ 503{,}600$$

$$D_{III} = -\ 74{,}900$$

C_{60} min.—

Minimum span = 86·8

$$\frac{d\,C}{d\,g} = 19\ W - 22\ W_3$$

Second driver of first engine on top hinge.

$$C_{60}\ min. = -\ 515{,}500$$

$$D_{IV} = -\ 25{,}500.$$—

Horizontal Component D of the Stresses in the Diagonals—

$$D_x = C_x - C_{x-\Delta x} = \Delta\, C_x = \left(\frac{M}{h}\right)_x - \left(\frac{M}{h}\right)_{x-\Delta x}$$

Using the bottom chord, we have:

$$D_1 = C_{15} - C_0$$

but

$$C_0 = \left(\frac{M}{h}\right)_0 - H = -\ H$$

hence:

$$C_{15} = [0{\cdot}35\ W_1 g_1 + W_2 (15 - 0{\cdot}65\ g_2) - 0{\cdot}45\ W_3 g_3]\ \frac{1}{22{\cdot}2}$$

subtracting $22{\cdot}2\ H = [0{\cdot}37\ W_1 g_1 + W_2 (0 + 0{\cdot}37\ g_2) +$

$$+ 0{\cdot}37\ W_3 g_3]\ \frac{1}{22{\cdot}2}$$

$$D_1 = [0{\cdot}72\ W_1 g_1 + W_2 (15 - 0{\cdot}28\ g_2) - 0{\cdot}08\ W_3 g_3]\ \frac{1}{22{\cdot}2}$$

$$D_{II} = C_{30} - C_{15} = \Delta\, C_{30}$$

$$22{\cdot}2\ C_{30} = [5{\cdot}55\ W_1 g_1 + W_2 (666{\cdot}0 - 16{\cdot}65\ g_2) -$$

$$- 7{\cdot}77\ W_3 g_3]\ \frac{1}{22{\cdot}2 \times 13{\cdot}8}$$

subtracting $13{\cdot}8\ C_{15} = [4{\cdot}83\ W_1 g_1 + W_2 (207{\cdot}0 - 8{\cdot}97\ g_2) -$

$$- 6{\cdot}21\ W_3 g_3]\ \frac{1}{22{\cdot}2 \times 13{\cdot}8}$$

$$D_{II} = [0{\cdot}72\ W_1 g_1 + W_2 (149{\cdot}0 - 7{\cdot}68\ g_2) -$$

$$- 1{\cdot}56\ W_3 g_3] \frac{1}{22{\cdot}2 \times 13{\cdot}8}$$

$$D_{III} = C_{45} - C_{80} = \Delta C_{45}$$

$$13{\cdot}8\ C_{45} = [2{\cdot}07\ W_1 g_1 + W_2 (621{\cdot}0 - 11{\cdot}73\ g_2) -$$

$$- 3{\cdot}45\ W_3 g_3] \frac{1}{13{\cdot}8 \times 7{\cdot}8}$$

subtracting $7{\cdot}8\ C_{80} = [1{\cdot}95\ W_1 g_1 + W_2 (234{\cdot}0 - 5{\cdot}85\ g_2) -$

$$- 2{\cdot}73\ W_3 g_3] \frac{1}{13{\cdot}8 \times 7{\cdot}8}$$

$$D_{III} = [0{\cdot}12\ W_1 g_1 + W_2 (387{\cdot}0 - 5{\cdot}88\ g_2) -$$

$$- 0{\cdot}72\ W_3 g_3] \frac{1}{13{\cdot}8 \times 7{\cdot}8}$$

$$D_{IV} = C_{80} - C_{45} = \Delta C_{80}$$

$$7{\cdot}8\ C_{80} = [0{\cdot}39\ W_1 g_1 + W_2 (468{\cdot}0 - 7{\cdot}41\ g_2) -$$

$$- 1{\cdot}17\ W_3 g_3] \frac{1}{7{\cdot}8 \times 4{\cdot}2}$$

subtracting $4{\cdot}2\ C_{45} = [0{\cdot}63\ W_1 g_1 + W_2 (189{\cdot}0 - 3{\cdot}57\ g_2) -$

$$- 1{\cdot}05\ W_3 g_3] \frac{1}{7{\cdot}8 \times 4{\cdot}2}$$

$$D_{IV} = [- 0{\cdot}24\ W_1 g_1 + W_2 (279{\cdot}0 - 3{\cdot}84\ g_2) -$$

$$- 0{\cdot}12\ W_3 g_3] \frac{1}{7{\cdot}8 \times 4{\cdot}2}$$

$$D_V = C_{75} - C_{10} = \Delta C_{75} = \left[\left(\frac{M}{h}\right)_{75} - H\right] - C_{80} =$$

$$= - [H + C_{80}]$$

$$C_{80} = [0{\cdot}05\ W_1 g_1 + W_2 (60{\cdot}0 - 0{\cdot}95\ g_2) - 0{\cdot}15\ W_3 g_3] \frac{1}{4{\cdot}2}$$

adding $4{\cdot}2\ H = [0{\cdot}07\ W_1 g_1 + W_2 (\rho + 0{\cdot}07\ g_2) + 0{\cdot}07\ W_3 g_3] \frac{1}{4{\cdot}2}$

$$D_V = - [0{\cdot}12\ W_1 g_1 + W_2 (60 - 0{\cdot}88\ g_2) - 0{\cdot}08\ W_3 g_3] \frac{1}{4{\cdot}2}$$

Check:

Using the top chord we find:

$$D_I = C_{15} - C_0 = \left(\frac{M}{h}\right)_{15} - \left(\frac{M}{h}\right)_0 = \left(\frac{M}{h}\right)_{15}$$

$$D_I = [0{\cdot}72\ W_1 g_1 + W_2 (15 - 0{\cdot}28\ g_2) - 0{\cdot}88\ H_3 g_3] \frac{1}{22{\cdot}2}$$

$$D_I = C_{30} - C_{15}\ .$$

$$22{\cdot}2\ C_{30} = [10{\cdot}656\ W_1 g_1 + W_2 (666{\cdot}0 - 11{\cdot}544\ g_2) -$$

$$- 2{\cdot}064\ W_3 g_3] \frac{1}{22{\cdot}2 \times 13{\cdot}8}$$

subtracting $13{\cdot}8\ C_{15} = [9{\cdot}936\ W_1 g_1 + W_2 (207{\cdot}0 - 3{\cdot}864\ g_2) -$

$$- 1{\cdot}104\ W_3 g_3] \frac{1}{22{\cdot}2 \times 13{\cdot}8}$$

$$D_{II} = [0{\cdot}72\ W_1 g_1 + W_2 (459{\cdot}0 - 7{\cdot}68\ g_2) -$$

$$- 1{\cdot}56\ W_3 g_3] \frac{1}{22{\cdot}2 \times 13{\cdot}8}$$

$$D_{III} = C_{45} - C_{30} = [18{\cdot}3\ C_{45} - 7{\cdot}8\ C_{30}] \frac{1}{13{\cdot}8 \times 7{\cdot}8} =$$

$$= [0{\cdot}12\ W_1 g_1 + W_2 (387{\cdot}0 - 5{\cdot}88\ g_2) -$$

$$- 0{\cdot}72\ W_3 g_3] \frac{1}{13{\cdot}8 \times 7{\cdot}8}$$

$$D_{IV} = C_{60} - C_{45} = [7{\cdot}8\ C_{60} - 4{\cdot}2\ C_{45}] \frac{1}{7{\cdot}8 \times 4{\cdot}2} =$$

$$= [-0{\cdot}24\ W_1 g_1 + W_2 (279{\cdot}0 - 3{\cdot}84\ g_2) -$$

$$- 0{\cdot}12\ W_3 g_3] \frac{1}{7{\cdot}8 \times 4{\cdot}2}$$

$$D_V = C_{75} - C_{60} = -C_{61} = -[0{\cdot}12\ W_1 g_1 + W_2 (60 - 0{\cdot}88\ g_2) -$$

$$- 0{\cdot}08\ W_3 g_3] \frac{1}{4{\cdot}2}$$

Maxima:

$$D_I = C_{15}\ max. = \left(\frac{M}{h}\right)_{15} max. = 62{,}600 \text{ pounds.}$$

$D_{II}\ max.$ — Max. line $— g_m = \frac{45{,}900}{768} = 59{\cdot}64$ from $b = 0$

$$\text{Max. span} = a + (g_2 - a) = 59{\cdot}64$$

$$\frac{d\,D}{d\,g} = -\,a\,(W_1 + w) + \beta\,W_2 + \frac{w}{\Delta x}\,[a\,x - (k - \beta\,x)] =$$

$$= -\,0{\cdot}72\,(W_1 + w) + 7{\cdot}68\,W_2 - 207\,\frac{w}{15}$$

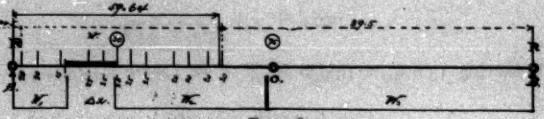


FIG. 18.

To the right of the dividing line (15)

$$[W_1 = 55\ w = 75]\ W_2 = 130$$

To the left of the dividing line (15)

$$[W_1 = 55\ w = 50]\ W_2 = 155$$

whence

$$\frac{d\,D}{d\,g} = \begin{cases} -\,75{\cdot}6 + 1190{\cdot}0 - 690{\cdot}0 = -\,765{\cdot}6 + 1190{\cdot}0 > o \\ -\,93{\cdot}6 + 998{\cdot}0 - 1035{\cdot}0 = -\,1128{\cdot}6 + 998{\cdot}0 < o \end{cases}$$

and

$$R\,l = 30{\cdot}617$$

$$R'\,l = 8{,}383$$

Check:

$$R\,l + R'\,l = 39{,}000 = 260 \times 150 = W\,l$$

$$R_{15} = 452$$

$$R_{30} = 1{,}596$$

$$R_{75} = 11{,}117$$

$$H\,f = R\,\frac{l}{2} - R_{75} = 4{,}192$$

Check :

$$R'_{15} = 8{,}383$$

$$R'_{30} = 4{,}935$$

$$R'_{75} = o$$

$$H f = R' \frac{l}{2} - R'_{75} = 4{,}192$$

$$H y_{15} = \left(H f . \frac{y}{f}\right)_{15} = 4{,}192 \times 0{\cdot}36 = 1{,}509$$

$$H y_{30} = H f \times 0{\cdot}64 = 2{,}682$$

$$\left(\frac{M}{h}\right)_{15} = [R\, l \frac{x}{l} - R_{15} - H y_{15}] \frac{1}{h} = [R' l \frac{x}{l - x} - R'_{15} -$$

$$- H y_{15}] \frac{1}{h} = 50{,}000$$

$$\left(\frac{M}{h}\right)_{30} = [R l \frac{x}{l} - R_{30} - H y_{30}) \frac{1}{h} = [R' l \frac{x}{l - x} - R'_{30} -$$

$$- H y_{30}] \frac{1}{h} = 133{,}700$$

$$D_{II} = \left(\frac{M}{h}\right)_{30} - \left(\frac{M}{h}\right)_{15} = + 83{,}700 \text{ pounds.}$$

D_{III} *max.*—

Maximum line = 65·82

Maximum span = 65·82

$$\frac{d D}{d g} = - 0{\cdot}12 (W_1 + w) + 5{\cdot}88 W_2 - 117 \frac{w}{\Delta x}$$

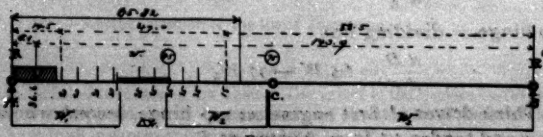



FIG. 19.

$$D_{III} = \left(\frac{M}{h}\right)_{45} - \left(\frac{M}{h}\right)_{30} = 233{,}000 - 125{,}900 =$$

$$= 107{,}100 \text{ pounds.}$$

D_{IV} *max.*—

Maximum line = 72·66

Maximum span = $(g_m - a)$ = from (60) to 72·56

a and c being negative.

$$\frac{d\,D}{d\,g} = + 0{\cdot}24\ w + 3{\cdot}84\ W_2 - 63\ \frac{w}{dx}$$

Third driver of first engine on (60), last wheel of tender of first engine 59·5 from Abutment B.

$$\frac{d\,D}{d\,g} = \begin{cases} 0{\cdot}24 \times 65 + 3{\cdot}84 \times 75 - 0{\cdot}12 \times 80 - \frac{50}{15} \times 63 = \\ \qquad = 3{,}036 - 219{\cdot}6 > o \\ 0{\cdot}24 \times 90 + 3{\cdot}84 \times 50 - 0{\cdot}12 \times 80 - \frac{75}{15} \times 63 = \\ \qquad = 213{\cdot}6 - 324{\cdot}6 < o \end{cases}$$

$$D_{IV} = \left(\frac{M}{h}\right)_{60} - \left(\frac{M}{h}\right)_{45} = + 55{,}200 - (- 30{,}000) = + 85{,}200$$

D_V *max.*—

$$D_V = \left(\frac{M}{h}\right)_{75} - \left(\frac{M}{h}\right)_{60} = - \left(\frac{M}{h}\right)_{60} = - 260{,}600 \text{ pounds.}$$

Minima.—

$$D_I = \left(\frac{M}{h}\right)_{15} - \left(\frac{M}{h}\right)_{0} = \left(\frac{M}{h}\right)_{15} \textit{ min.} = - 56{,}400 \text{ pounds.}$$

D_{II} *Min.*—

Min. span = 90·36

$$\frac{d\,D}{d\,g} = 7{\cdot}68\ W_2 - 1{\cdot}56\ W_3$$

top hinge = dividing line

$$\frac{d\,D}{d\,g} = 64\ W - 77\ W_3$$

Third driver of first engine on top hinge, second wheel of second tender 1·5 from Abutment B.

$$D_{II}\ \textit{min.} = \left(\frac{M}{h}\right)_{90} - \left(\frac{M}{h}\right)_{15} = - 132{,}000 + 56{,}400 =$$

$$= - 75{,}600$$

D_{III} *Min.*—

$$\text{Min. span} = 84{\cdot}18$$

$$\frac{d\,D}{d\,g} = 49\ W - 55\ W_3$$

Second driver of first engine on top hinge, first wheel of second tender 1·75 from Abutment *B*.

$$D_{III}\ min. = \left(\frac{M}{h}\right)_{45} - \left(\frac{M}{h}\right)_{30} = -\ 223{,}600\ +\ 130{,}600 =$$

$$= -\ 93{,}000 \text{ pounds.}$$

D_{IV} *Min.*—

$$\text{Max. line } g_m = 72{\cdot}6$$

$$\text{Min. span} = a + \left(\frac{1}{2} - g_m\right) + c$$

or from *o* to (45) and from (72·6) to (150)

$$\frac{d\,D}{d\,g} = -\ a\ (W_1 + w) + \beta\ W_3 + c\ W_2 +$$

$$+ \frac{w}{d\,x}\left[a\,x - (k - \beta\,x)\right]$$

$$\frac{d\,D}{d\,g} = 0{\cdot}24\ (W_1 + w)\ 3{\cdot}84 + W_2 - 0{\cdot}12\ W_3 - 4{\cdot}2\ w$$

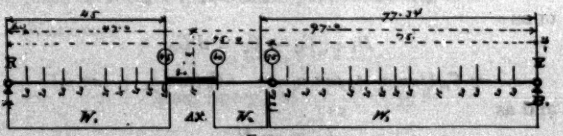


FIG. 20.

Top hinge = dividing line. To be a maximum $W_1\,g_1$ must have a load in (45).

[*Note.*—Another position of the loads answering

$$\frac{d\,D}{d\,g}$$

can be found in reversing the trains. The values of the stresses corresponding to these positions must be calculated and the absolute maximum found. I have in all instances only given the latter.]

$$D_{IV} = \left(\frac{M}{h}\right)_{90} - \left(\frac{M}{h}\right)_{45} = -\ 54{,}900 - (+\ 12{,}400) =$$

$$= -\ 67{,}300.$$

D_V *min.*—

$$D_V = \left(\frac{M}{h}\right)_{75} - \left(\frac{M}{h}\right)_{10} = -\left(\frac{M}{h}\right)_{10} = +\ 258{,}600$$

Vertical Stresses P in the Posts.

For determining P maximum and P minimum always that end of the post must be selected taking up no moving loads.

(I) *One diagonal acting at the end of the post.*

(*a*) *Diagonals running from left to right corresponding to* $+ D$

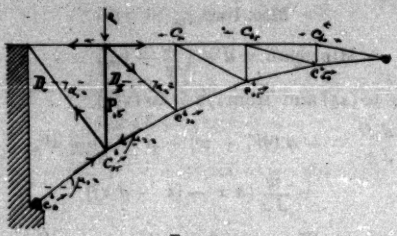


FIG. 21.

$$P_{15} = C_0 \tan \mu_1 - C_{15} \tan \mu_2 + D_1 \tan a_1$$

or as:

$$D_1 = C_{15} - C_0 = \Delta C_{15}$$

$$= C_{15} (\tan a_1 + \tan \mu_2 =) - C_0 (\tan a_1 + \tan \mu_1)$$

and as

$$C_0 = -H$$

$$P_{15} = 2{\cdot}04\ C_{15} + 2{\cdot}20\ H$$

$$2{\cdot}04\ C_{15} = [0{\cdot}714\ W_1 g_1 + W_2 (30{\cdot}6 - 1{\cdot}326\ g_2) -$$

$$- 0{\cdot}918\ W_3 g_3] \frac{1}{22{\cdot}2}$$

$$2{\cdot}20\ H = [0{\cdot}814\ W_1 g_1 + W_2 (0 + 0{\cdot}814\ g_2) +$$

$$+ 0{\cdot}814\ W_3 g_3] \frac{1}{22{\cdot}2}$$

$$P_{15} = [1{\cdot}528\ W_1 g_1 + W_2 (30{\cdot}6 - 0{\cdot}512\ g_2) - 0{\cdot}104\ W_3 g_3] \frac{1}{22{\cdot}2}$$

$$P_{30} = C_{15} \tan \mu_2 - C_{30} \tan \mu_3 + D_{II} \tan \alpha_2$$

or as:

$$D_{II} = C_{30} - C_{15} = \Delta C_{30}$$

$$= C_{30} (\tan \alpha_2 + \tan \mu_3) - C_{15} (\tan \alpha_2 + \tan \mu_2)$$

$$= 1{\cdot}32\ C_{30} - 1{\cdot}48\ C_{15}$$

$$= [0{\cdot}1776\ W_1 g_1 + W_2 (572{\cdot}76 - 8{\cdot}7024 g_2) -$$

$$- 1{\cdot}0656\ W_3 g_3] \frac{1}{22{\cdot}2 \times 13{\cdot}8}$$

$$P_{45} = 0{\cdot}76\ C_{45} - 0{\cdot}92\ C_{30}$$

$$= [-0{\cdot}2208\ W_1 g_1 + W_2 (256{\cdot}68 - 3{\cdot}5328 g_2) -$$

$$- 0{\cdot}1104\ W_3 g_3] \frac{1}{13{\cdot}8 \times 7{\cdot}8}$$

$$P_{60} = 0{\cdot}36\ C_{60} - 0{\cdot}52\ C_{45}$$

$$= [-0{\cdot}1872\ W_1 g_1 + W_2 (70{\cdot}20 - 0{\cdot}8112 g_2) +$$

$$+ 0{\cdot}1248\ W_3 g_3] \frac{1}{7{\cdot}8 \times 4{\cdot}2}$$

From these general equations we deduct the maximum stresses of P corresponding to positive values of D.

Maxima.—

$$P_{15} \text{ max. } (+ D)$$

$$\text{Max. line } g_m = \frac{30{,}600}{512} = 59{\cdot}8$$

$$\text{Max. span} = a + (g_m - a) = 59{\cdot}8$$

As shown by

$$\frac{1}{22{\cdot}2}$$

P_{15} is a function of

$$\left(\frac{M}{h}\right)_{15}$$

only.

$$\frac{d P}{d g} = 1{,}528\ W_1 - 512\ W_2$$

$$= 255\ W_1 - 64\ W$$

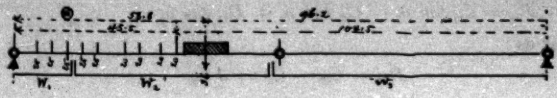


FIG. 22.

Dividing line point (15):

$$\frac{d\,P}{d\,g} = \begin{matrix} 15{,}424 - 19{,}125 < o \\ 15{,}424 - 12{,}750 > o \end{matrix}$$

and

$$R\ l = 29{,}278$$

$$R'\,l = 6{,}871$$

$$\text{Check } R\,l + R'\,l = 36{,}149 = 241 \times 150$$

$$R_{15} = 319$$

$$R_{30} = 1{,}875$$

$$R_{75} = 11{,}203\ H\,f = R\,\frac{l}{2} - R_{75} = 3{,}436$$

$$\text{Check } R'_{15} = 3{,}576$$

$$R'_{30} = 1{,}517$$

$$R'_{75} = o\ H\,f = R'\,\frac{l}{2} - R_{75} = 3{,}436$$

$$H\,f + f = H = 114{,}500$$

$$\left(\frac{M}{h}\right)_{30} = 129{,}100$$

$$\left(\frac{M}{h}\right)_{15} = 61{,}800$$

$$D_{\text{I}} = \left(\frac{M}{h}\right)_{15} - \left(\frac{M}{h}\right)_{o} = \left(\frac{M}{h}\right)_{15} = 61{,}800$$

$$C_o = H = 114{,}500$$

$$C_{15} = H - \left(\frac{M}{h}\right)_{15} = 52{,}700$$

$$P_{15} = C_o\ tan\ \mu_1 - C_{15}\ tan\ \mu_2 + D_{\text{I}}\ tan\ a_1$$

$$= 0{\cdot}72 \times 114{,}500 - 0{\cdot}56 \times 52{,}700 + 61{,}800 \times 1{\cdot}48 =$$

$$= 114{,}500$$

Check from top chord.

$$P_{15} = D_{II} \tan a_2 + p = (12{,}910 - 61{,}800) \times 0{\cdot}92 + p =$$
$$= 57{,}300 \times 0{\cdot}92 + 82{,}600 = 114{,}500$$

P_{30} *max.* $(+ D)$:

As seen from

$$\frac{1}{22{\cdot}2 \times 13{\cdot}8},$$

P_{30} is a function of

$$\left(\frac{M}{h}\right)_{30} - \left(\frac{M}{h}\right)_{15} \text{ and goes as } D.$$

Maximum span $= 65{\cdot}8$

$$\frac{dP}{dg} = -a\,(W_1 + w) + \beta\, W_2 + \frac{w}{\Delta x}\,[a\,x - (k - \beta x)]$$

$$= 0{\cdot}1776\,(W_1 + w) + 8{\cdot}7024\, W_2 - \frac{w}{15} \times 306{\cdot}33$$

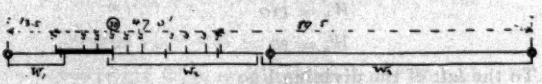


FIG. 23.

$$\frac{dP}{dg} = \begin{cases} -16{\cdot}0 + 1131{\cdot}3 - 1531{\cdot}6 = 1{,}131 - 1{,}547 < o \\ -11{\cdot}5 + 1348{\cdot}9 - 1021{\cdot}1 = 1{,}349 - 1{,}043 > o \end{cases}$$

$$H = 139{,}700$$

$$\left(\frac{M}{h}\right)_{15} = 49{,}700$$

$$\left(\frac{M}{h}\right)_{30} = 133{,}700$$

$$\left(\frac{M}{h}\right)_{45} = 216{,}400$$

$$D_{II} = \left(\frac{M}{h}\right)_{30} - \left(\frac{M}{h}\right)_{15} = 84{,}000$$

$$D_{III} = 82{,}700$$

$$C_{45} \tan \mu_3 = 50{,}400$$

$$C_{30} \tan \mu_2 = 24{,}000$$

$$P_{30} = C_{45} \tan \mu_3 - C_{30} \tan \mu_2 + D_{III} \tan \sigma_3 = 125{,}300$$

Check from top chord.

$$P_{30} = D_{III} \tan a_3 + p = 82{,}700 \times 0{\cdot}52 + 82{,}500 = 125{,}500$$

P_{45} *max.*—

$$\text{Maximum span} = (g_m - a) = 27{\cdot}7$$

or from (45) to (72·7).

$$\frac{dP}{dg} = -a\,(W_1 + w) + \beta\, W_2 + c\, W_3 + \frac{w}{\Delta x}\,[a\,x - (k - \beta x)]$$

$$= +\,0{\cdot}22\,(W_1 + w) + 3{\cdot}53\, W_2 - 0{\cdot}11\, W_3 - 107{\cdot}6\,\frac{w}{15}$$

Third driver of first engine in (45), last wheel of first tender 74·5 from Abutment *B*.

To the right of the dividing line

$$(W_1 + w) = 90$$
$$W_2 = 110$$
$$W_3 = 20$$

To the left of the divining line

$$(W_1 + w) = 65$$
$$W_2 = 135$$
$$W_3 = 20$$

$$\frac{dP}{dg} = \begin{cases} 14{\cdot}3 + 476{\cdot}5 - 2{\cdot}2 - 359{\cdot}0 = 490{\cdot}8 - 361{\cdot}2 > o \\ 19{\cdot}8 + 383{\cdot}3 - 2{\cdot}2 - 538{\cdot}0 = 403{\cdot}1 - 540{\cdot}2 < o \end{cases}$$

$$P_{45} = C_{45} \tan \mu_3 - C_{30} \tan \mu_2 + D_{III} \tan a_3 = D_{IV} \tan a_4 + p =$$

$$= 93{,}500.$$

P_{60} *max.*—

$$\text{Maximum line } g_m = 86{\cdot}6$$

Maximum line from (60) to end of truss (150).

$$\frac{dP}{dg} = -a\,(w) + \beta\, W_1 + c\, W_2 + \frac{w}{\Delta x}\,[a\,x - (k - \beta x)]$$

$$= +\,0{\cdot}187\,(w) + 0{\cdot}811\, W_2 + 0{\cdot}125\, W_3 - 32{\cdot}7\,\frac{w}{\Delta x}$$

First wheel 47·75 from Abutment *A*, second wheel of second tender, 1·25 from Abutment *B*.

$$P_{80} = C_{45} \tan \mu_3 - C_{80} \tan \mu_4 + D_{IV} \tan a_4 = D_V \tan a_5 + p =$$
$$= 98{,}100 \text{ pounds.}$$

(*b*) *Diagonals running from the right to the left corresponding to minus D.*

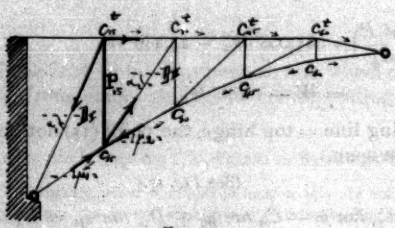


FIG. 24.

$$P_{15} = C_{30} (\tan a''_2 - \tan \mu_2) - C_{15} (\tan a''_2 - \tan \mu_1)$$
$$= 0{\cdot}92\ C_{30} - 0{\cdot}76\ C_{15}$$
$$= [1{\cdot}435\ W_1 g_1 + W_2 (455{\cdot}40 - 8{\cdot}5008\ g_2) -$$
$$- 2{\cdot}4288\ W_3 g_3\ ] \frac{1}{22{\cdot}2 \times 13{\cdot}8}$$

$$P_{30} = C_{45} (\tan a''_3 - \tan \mu_3) - C_{30} (\tan a''_3 - \tan \mu_2$$
$$= 0{\cdot}52\ C_{45} - 0{\cdot}36\ C_{30}$$
$$= [0{\cdot}3774\ W_1 g_1 + W_2 (238{\cdot}68 - 3{\cdot}9936\ g_2) -$$
$$- 0{\cdot}8112\ W_3 g_3\ ] \frac{1}{13{\cdot}8 \times 7{\cdot}8}$$

$$P_{45} = C_{60} (\tan a''_4 - \tan \mu_4) - C_{45} (\tan a''_4 - \tan \mu_3)$$
$$= 0{\cdot}24\ C_{60} - 0{\cdot}12\ C_{45}$$
$$= [0{\cdot}3360\ W_1 g_1 + W_2 (108{\cdot}36 - 1{\cdot}6464\ g_2) -$$
$$- 0{\cdot}2016\ W_3 g_3] \frac{1}{7{\cdot}8 \times 4{\cdot}2}$$

From these general equations we deduct the maximum stresses corresponding to negative values of *D*.

P_{15} *max.* (— D). —

$$\text{Max. line} = \frac{544.40}{8.5008} = 53.5$$

Max. span = minimum span of — D

$$= \left(\frac{l}{2} - g_m\right) + c = 96.5$$

$$\left(\frac{d\,P}{d\,g}\right) = 85{,}008\ W_2 - 24{,}288\ W_3$$

$$= W - 1.286\ W_3$$

Dividing line = top hinge, the point (15) not being in the maximum span.)

(See *Fig. 17.*)

$$P_{15} = C_{15}\ tan\ \mu_1 - C_{30}\ tan\ \mu_2 + D_{II}\ tan\ a_{II} = D_I\ tan\ a_1 = 124{,}100$$

The above position of the loads for P_{15} maximum corresponds to the position of loads making D_I a minimum.

From the general equation follows, that stress P_{15} is a function of D_{II}, but in the case of $p = o$, or when no moving load is taken up in the panel point, P_{15} becomes a function of D_I, the above position of the loads corresponding to D_I min., showing how difficult it would be to determine the correct position of the loads, inducing the maximum or minimum stress, if p had to be taken into consideration and how necessary it is to eliminate p in defining the maximum or minimum of the stress.

P_{30} *max.* (— D)—

$$\text{Max. line } g_m = 59.7$$

$$\text{Max. span} = \left(\frac{l}{2} - g\right) + c = 90.3$$

$$\left(\frac{d\,P}{d\,g}\right) = W - 1.2\ W_3$$

Position of loads as for D_{II} min. (see D_{II} min.).

$$P_{30} = D_{II}\ tan\ a''_2 = 1.48 + 75{,}500 = 111{,}800$$

P_{45} *max.* $(-D)$.—

$$\text{Max. line } g_m = 65{\cdot}68$$

$$\text{Max. span} = \left(\frac{l}{2} - g_m\right) + c = 84{\cdot}32$$

$$\frac{dP}{dg} = 16{,}464\ W_2 - 2{,}016\ W_3$$

$$= W - 1{\cdot}723\ W_3$$

First wheel 62·75 from Abutment A, first wheel of second tender 0·75 from same abutment, corresponding to D_{III} minimum.

$$P_{45} = D_{III}\ tan\ a''_3 = 0{\cdot}92 \times 93{,}000 = 85{,}600 \text{ pounds.}$$

[*Note.*—It is necessary, when diagonals in tension only are acting, to compute, in *every instance*, the general equations of P corresponding to minus D, as by no means can be foretold whether plus D or minus D will produce the greatest stresses in the posts.]

(II) *Two Diagonals Acting Together at the End of the Post.*

The stresses from two diagonals in tension acting together at the end of the post, are, as we know, smaller than those from one diagonal, and no further investigations are necessary.

(III) *No Diagonal Acting at the End of the Post.*

As known, the stress is smaller than the vertical component of the maximum or minimum chord stress.

Maximum:

$$P_{15} = C_{15}\ min. \times 0{\cdot}16 = -64{,}200$$

$$P_{10} = C_{80}\ min. \times 0{\cdot}16 = -71{,}000$$

$$P_{45} = C_{45}\ min. \times 0{\cdot}16 = -80{,}600$$

$$P_{80} = C_{80}\ min. \times 0{\cdot}16 = -82{,}500$$

$$tan\ \mu_{x+\Delta x} - tan\ \mu_x = 0{\cdot}16$$

for all the points.

These stresses being smaller than those from one diagonal, no further investigation is necessary.

Minimum.—With two *diagonals in tension* at the end of the posts, it is evident that no tensile stress can occur in the post.

Hence P_{15}, P_{30}, P_{45} are exempt of tensile stresses, not so P_{60}.

P_{60}.—Remembering that the bottom chord is deciding the maximum and minimum stress of P, we see that P_{60} can have only one diagonal or no diagonal acting at its end.

The latter position is in so far different from the other posts, as D_V can be also in compression.

Beginning with *Tension* we have, to start with, the position of C_{60} maximum, which chances to correspond without further shifting to P_{60} maximum, as D_{IV} is negative,

$$D_V = o$$

and

$$C_{60}\,(tan\ \mu_5 - tan\ \mu_4)$$

the greatest value possible.

Hence

$$P_{60}\ min. = 96{,}600 \times 0{\cdot}16 = 15{,}500 \text{ tension.}$$

Check from top chord.

$$P_{60} = D_V\ tan\ a_5 + D_{IV}\ tan\ a_4 + p = 232{,}400 \times 0{\cdot}2 + 700 \times$$
$$\times\ 0{\cdot}52 + 31{\cdot}200 = 15{,}600$$

Compression:

Starting with C_{60} min., we find D_{IV} positive. Moving to the left we find the position corresponding to $D_{IV} = o$ thus:

FIG. 25.

and

$$P_{60} = 503{\cdot}800 \times 0{\cdot}16 = 80{,}600$$

Check from top chord.

$$P_{60} = D_V\ tan\ a_5 + p = 217.700 \times 0{\cdot}2 + 36{,}800 = 80{,}400$$

In the same way can also be found the correct values of P maximum and P minimum, when two diagonals in tension are acting.

With these data the actual stresses in the members are calculated as follows:

Chord Members:

The *direction* of the stress in the chord members, *which cannot be foretold,* is deducted from the position of the diagonals in the panels. For this reason the horizontal component of the stresses in the diagonal or D, corresponding to the position of the loads producing the maximum or minimum stress in the chord members, have been calculated (see chord stresses). Computing these values of D we find the direction of the stresses in the chords, as shown in *Fig. 26* and *Fig. 27*.

Calling μ the angle of the member with the horizontal, the stress in the inclined chord becomes

$$C_x \div \cos \mu_x$$

Top Chord.—

Maxima (positive moments, top chord in compression):

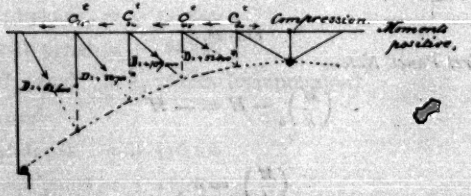


FIG. 26.

First panel = 62,600

Second panel = 137,300

Third panel = 233,100

Fourth panel = 260,600

Minima (negative moments, top chord in tension):

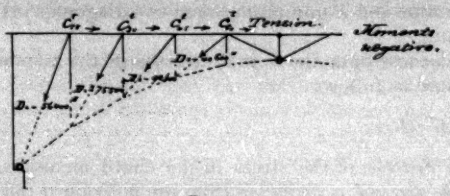


FIG. 27.

First panel = 0

Second panel = 56,400

Third panel = 131,900

Fourth panel = 223,500

Bottom Chord.—

Maxima (positive moments, bottom chord in tension):

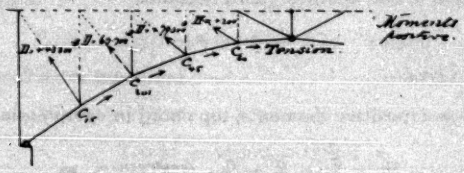


FIG. 28.

First Panel Note.—

$$\left(\frac{M}{h}\right)_0 - H = -H$$

as

$$\left(\frac{M}{h}\right)_0 = 0$$

Hence max. = min. $H = 0$

Second panel = + 16,700

Third panel = + 58,400

Fourth panel = + 111,400

Fifth panel = + 96,900

Minima (negative moments, bottom chord in compression):

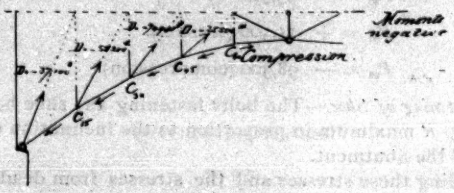


FIG. 29.

First panel = 495,200
Second panel = 509,000
Third panel = 542,700
Fourth panel = 530,000
Fifth panel = 517,000

Diagonals.—The stress in the inclined member will be

$$D \div \cos a$$

Maxima:

First panel = + 112,000
Second panel = + 113,700
Third panel = + 121,000
Fourth panel = + 88,500
Fifth panel = — 265,600 (compression)

Minima:

First panel = + 136,300
Second panel = + 135,200
Third panel = + 126,300
Fourth panel = + 75,900
Fifth panel = 263,600 (tension).

Posts:

$$P_{15} = -114{,}500$$

$$P_{30} = - 125{,}300$$

$$P_{45} = - 93{,}500$$

$$P_{60} = + 15{,}500 \text{ (tension).}$$

$$P_{60} = - 98{,}300 \text{ (compression).}$$

Fastening of Shoe.—The bolts fastening the shoe have to take up R maximum in proportion to the inclination of the seat of the abutment.

Adding these stresses and the stresses from dead load, we find the following strain sheet for *one truss*, in dividing the figures by 2.

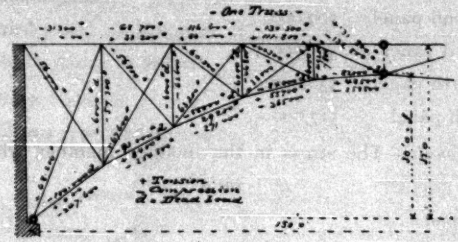


FIG. 30.

[*To be continued.*]

PROCEEDINGS

OF THE

CHEMICAL SECTION

OF THE

FRANKLIN INSTITUTE.

[*Proceedings of the stated meeting, held Tuesday, November 15, 1892.*]

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, November 15, 1892.

Dr. Wm. H. Wahl, President, in the chair.

Dr. Jayne presented a report on behalf of the special committee appointed to arrange for a reception to be given to Professor Wiley, on the evening of December 6, 1892.

On motion, the report was accepted and the committee was given power to arrange the necessary details.

Nominations for officers for the ensuing year were then made as follows:

For *President*,	Prof. Edgar F. Smith, Dr. Wm. H. Greene.
For *Vice-Presidents* (two to be elected),	Mr. Henry Bower, Dr. D. K. Tuttle.
For *Secretary*,	Dr. Wm. C. Day.
For *Treasurer*,	Dr. H. W. Jayne.
For *Conservator*,	Dr. Wm. H. Wahl.

Mr. H. Pemberton, Jr., on behalf of himself and Mr. George P. Tucker, read a paper on "Deposits of Native Soda near Laramie, Wyoming." Referred for publication.

Dr. Wm. H. Wahl read a paper, entitled "Observations on Ferro-Tungsten." Both papers called forth considerable discussion.

The Section then adjourned.

H. PEMBERTON, JR., *Secretary pro tem.*

REPORT OF THE PROCEEDINGS OF THE NINTH ANNUAL MEETING OF OFFICIAL AGRICULTURAL CHEMISTS, HELD AT WASHINGTON, AUGUST 25–27, 1892, WITH SPECIAL REFERENCE TO THE METHODS ADOPTED FOR THE CONTROL OF THE FERTILIZER TRADE.

By Dr. Bruno Terne.

[*Read at the stated meeting of the Chemical Section, held October 18, 1892.*]

Mr. President and Members of the Chemical Section:

Looking over the wide field of scientific societies, and more especially over the chemical organizations of our country, I know of none which has, in the short time since its creation, done so much practical good work for the benefit of the interested circles as the Association of Official Agricultural Chemists.

Still, when we come to criticise its work of the last few years, we cannot deny the fact that the results gained are not in proportion to the work done Although encouraging, the results are far from being final.

A short historical sketch of the association will doubtless interest you, and serve as an introduction to the remarks which I propose to make in regard to the results of the last meeting of the said association.

With the revival of business in the Southern States the consumption of fertilizing materials for the cotton plantations soon assumed enormous proportions.

The valuation of this material, based on chemical analysis, was a source of great trouble both to seller and buyer, caused by the unconcerted action of the chemical laboratories.

There were in use all possible methods for the determination of the different ingredients.

This chaotic state of affairs has been changed by the concerted action of the analytical chemists interested in such work, and the gain of this very important point is the fruit of the earnest work of this association.

The beginning of the organization dates from the following circular-letters:

DEPARTMENT OF AGRICULTURE,
ATLANTA, Ga., May 20, 1880.

DEAR SIR:—The experience of the last fertilizer season has suggested to my mind the importance of securing such uniformity of method in determining, by chemical analysis, the percentage of valuable ingredients in commercial fertilizers, as will give more uniform, and hence more satisfactory results. This is especially desirable in determining reverted phosphoric acid.

With a view to accomplishing so desirable an object, as well as others which may be deemed proper, I have the honor to suggest the calling of a convention of the several Commissioners of Agriculture, representatives from State Boards of Agriculture, State chemists and professors of chemistry in State universities and State agricultural colleges, in those States using large amounts of commercial fertilizers, to meet at some convenient point early in the month of July next.

I would be glad to have your views on the subject; and if favorable to the suggestion please nominate some gentleman who, by general assent, may be informally authorized to fix the time and place of such convention, and issue the necessary notices. You are also requested to give me the names and post-offices of gentlemen in your State, holding either of the above positions indicated, which do not appear in the list below, that copies of this circular may be sent to them. An early reply is desirable.

Very respectfully,
J. T. HENDERSON,
Commissioner of Agriculture.

DEPARTMENT OF AGRICULTURE,
ATLANTA, Ga., July 1, 1880.

DEAR SIR:—I am gratified to announce that the recent circular-letter which I had the honor to issue, suggesting the calling of a convention for the purpose of adopting a

uniform system for the analysis of commercial fertilizers, has met with favorable responses from a large majority of the gentlemen to whom it was sent. A like majority has imposed upon me the duty of fixing the time and place of said convention, and issuing the necessary notices for the same. After correspondence with others, and due consideration of the interests involved, I have decided upon Wednesday, the 28th of July, as the time, and Washington, D. C., as the place for the assembling of the proposed convention. You are, accordingly, respectfully and earnestly invited to be present and participate in the convention.

Every reasonable facility for the deliberations of the convention will be afforded by Hon. Wm. G. LeDuc, Commissioner of Agriculture, who is in hearty sympathy with the object sought to be accomplished. I have appended hereto a list of the names of gentlemen to whom this circular will be sent.

Trusting that you will find it convenient to attend and give the convention the benefit of your experience, and requesting that you will at once inform me by letter whether you will attend, I am, Respectfully,

J. T. HENDERSON,
Commissioner of Agriculture.

The first convention met on the day set for and adopted methods for the determination of phosphoric acid, nitrogen and potash to be used during the ensuing year.

This meeting was followed by meetings in Cincinnati, Atlanta, and in 1884 in Philadelphia.

The convention in our city was the most important one for the organization, because the association adopted (September 9th) the new Constitution, and made itself entirely independent of its former connections as a subdivision of the Section of Chemistry of the American Society for the Advancement of Science,

The first two paragraphs of the Constitution are of general interest.

(1) "This association shall be known as the Association of Official Agricultural Chemists in the United States. Its

object shall be to secure, as far as possible, uniformity in legislation with regard to the regulation of the sale of commercial fertilizers in the different States and uniformity and accuracy in the methods and results of fertilizer analysis.

(2) "Analytical chemists connected with Departments of Agriculture, State Agricultural Experiment Stations and State Boards, exercising an official fertilizer control, shall alone be eligible to membership; and one such representative from each of these institutions, when properly accredited, shall be entitled to a vote in the association. All analytical chemists and others interested in the objects of the association may attend its meetings and take part in its discussions, but shall have no vote in the association."

The exclusion of all chemists not to be classified as officials, from full membership, varies from the custom of England and Germany.

It is not my intention to question either the motive or the wisdom of this regulation, or of another rule denying the right to enter a motion which has been in force since 1891, as it certainly gives the excluded more liberty to criticise than if he were made responsible by his vote for the action of the society.

Since the Philadelphia meeting the society has made its headquarters in Washington, where, under the patronage of the Department of Agriculture, the annual meetings are held. The untiring efforts of the Secretary of the society, Mr. Harvey W. Wiley, Chief Chemist of the Department of Agriculture, have secured to the association the support of the Administration, and enabled the same to publish the results of their annual work, as a *Bulletin* of the Department, division of chemistry.

These annual reports are edited by the able pen of Professor Wiley and are an excellent guide for all connected with the interests of the society.

The scope of work of the association has extended to a wider field than was originally planned, including at present all station work in the interest of the farming community.

Besides the fertilizers analysis, we have had during the last year reports on analysis of fermented liquors, dairy

DETERMINATION OF POTASH BY THE LINDO-GLADDING METHOD WITH AND WITHOUT SODIUM CHLORIDE.

Names of Analysts and Location. Official Laboratories.	Sample No. 1.		Sample No. 2.		Sample No. 3.		Sample No. 4.	
	With NaCl.	Without NaCl.	With NaCl.	Without NaCl.	With NaCl.	Without NaCl.	With NaCl.	Without NaCl.
Norman Robinson, State Chemist, Tallahassee, Fla.,	48·83	48·70	1·58	1·59	1·47	1·46	1·40	1·41
F. S. Shiver, Assistant of State Chemist, Fort Hill, S. C. (*a*),	49·05	49·13	1·45	1·44	1·40	1·39	1·40	1·44
F. S. Shiver, Assistant of State Chemist, Fort Hill, S. C. (*b*),	49·00	—	1·45	—	1·40	—	1·39	—
P. L. Hutchinson, Assistant of State Chemist, Atlanta, Ga.,	49·05	49·15	1·50	1·48	1·51	1·40	1·48	1·40
J. F. Wilkinson, Assistant of State Chemist, Atlanta, Ga.,	49·13	49·12	1·50	1·52	1·47	1·42	1·36	1·34
F. B. Carpenter, Experiment Station, Raleigh, N. C.,	48·94	—	1·61	—	1·48	—	1·56	—
Thos. L. Blalock, Experiment Station, Raleigh, N. C. (*a*),	48·76	48·76	1·63	1·63	1·48	1·46	1·48	1·47
Thos. L. Blalock, Experiment Station, Raleigh, N. C. (*b*),	48·96	49·10	—	—	—	—	—	—
Thos. L. Blalock, Experiment Station, N. C. (*c*),	48·90	48·91	1·59	1·58	1·52	1·48	1·46	1·45
Louis A. Voorhees, Experiment Station, New Brunswick, N. J.,	49·23	49·19	1·49	1·46	1·35	1·36	1·36	1·37
Louis A. Voorhees, Experiment Station, New Brunswick, N. J.,	49·27	49·29	1·51	1·52	1·38	1·42	1·36	1·37
Chas. S. Cathcart, Agricultural College, New Brunswick, N. J.,	49·31	49·34	1·52	1·46	1·43	1·45	1·35	1·33
E. H. Farrington, Experiment Station, Champaign, Ill.,	49·20	49·00	1·77	1·64	1·77	1·66	1·62	1·51
Thos. L. Watson, Experiment Station, Blacksburg, Va., Assistant Chemist,	49·10	48·95	1·55	1·58	1·41	1·42	1·41	1·36
R. C. Kedzie, Agricultural College P. O., Mich.,	48·73	48·69	1·55	1·53	1·54	1·51	1·67	1·65
Rudolph de Roode, Experiment Station, Morgantown, W. Va.,	48·96	48·96	1·52	1·55	1·42	1·42	1·37	1·39
Rudolph de Roode, Experiment Station, Morgantown, W. Va.,	49·04	49·04	1·57	1·53	1·48	1·46	1·39	1·37
C. B. Collingwood, Experiment Station, Tucson, Ariz.	48·62	48·38	1·51	1·47	—	—	—	—
Henry E. Curtis, Experiment Station, Lexington, Ky.,	49·00	48·91	1·45	1·46	1·43	1·40	1·39	1·37
W. S. Sweetser, Experiment Station, State College P. O., Pa.,	49·26	49·28	1·69	1·57	1·58	1·53	1·45	1·43
W. S. Sweetser, Experiment Station, State College P. O., Pa.,	49·30	49·29	1·62	1·57	1·51	1·51	1·48	1·40
H. B. McDonnell, State Chemist, College Park P. O., Md.,	48·47	48·69	1·65	1·61	1·58	1·55	1·48	1·45
W. D. Bigelow, U. S. Department of Agriculture, Washington, D. C.,	48·56	48·31	1·49	1·58	1·57	1·37	1·47	1·42
Average,	48·994	48·961	1·554	1·538	1·484	1·456	1·439	1·417
Difference,	—	·033	—	·016	—	·028	—	·022
Unofficial Laboratories.								
Shepard Laboratory, Charleston, S. C.,	49·88	49·80	1·87	1·91	1·75	1·74	1·66	1·55
Elwyn Waller School of Mines, Columbia College, New York City,	49·67	49·17	1·60	1·46	1·61	1·53	1·59	1·39
Elwyn Waller School of Mines, Columbia College, New York City,	49·89	48·71	1·62	1·52	1·56	1·52	—	1·53
Elwyn Waller School of Mines, Columbia College, New York City,	—	48·21	—	—	—	—	—	—
Elwyn Waller School of Mines, Columbia College, New York City,	—	48·69	—	—	—	—	—	—
H. T. Vulte, Assistant of Elwyn Waller School of Mines, Columbia College, New York City,	48·88	48·54	1·66	1·58	1·58	1·58	1·58	1·52
B. C. Hinman, Assistant of Elwyn Waller School of Mines, Columbia College, New York City,	—	48·99	—	1·51	1·67	1·40	1·41	1·40
F. S. Hyde, Student of School of Mines, Columbia College, New York City,	49·43	49·47	1·35	1·42	—	—	—	—
Average,	49·550	48·948	1·620	1·566	1·634	1·544	1·560	1·518
Difference,	—	·602	—	·054	—	·090	—	·042
Total average,	49·093	48·957	1·566	1·545	1·543	1·474	1·458	1·438
Difference,	—	·136	—	·021	—	·039	—	·020
Highest,			49·89		1·87		1·77	1·67
Lowest,			48·62		1·35		1·35	1·32
Difference,			1·27		0·52		0·42	0·35

products, foods rich and poor in carbo-hydrates, and on sugar.

During the last four years I have regularly attended the meetings of the association as a non-member, and taken much interest in the analysis of commercial fertilizers.

It was with great pleasure that I attended the meetings from August 25th to 27th of this year.

I will now report the proceedings, referring for the methods in use to the official *Bulletin* No. 31, 1891:

REPORT ON POTASH.

For the Association of Official Agricultural Chemists of the United States.

By Dr. George F. Payne, Atlanta, Ga.

The work required by the association, of the reporter on potash for 1892, was an investigation as to whether the use of sodium chloride was essential in the Lindo-Gladding method of determining potash.

Four samples were prepared and were numbered one, two, three and four.

No. 1 was a high grade sulphate of potash.

No. 2 was an acid phosphate with sulphate of potash in the form of kainit.

No. 3 was an acid phosphate with sulphate of potash in the form of kainit, and containing five per cent. of double sulphate of alumina and ammonia.

No. 4 was identical with No. 3 with the substitution of five per cent. of soluble sodium silicate in place of the double salt of alumina and ammonia.

The ingredients of each sample were first finely powdered and thoroughly mixed, then sifted through a very fine sieve, then again intimately mixed and portions taken from forty different parts of the pile were well re-mixed and used for filling the bottles. Only the best XX superfine long taper corks were used, and were selected long enough to project well from the mouth of the vial to secure easy opening and air-tight re-corking. The corks were all well sealed with paraffine. With each package a circular was sent requesting that the determinations be made as soon as convenient after receiving samples to eliminate as far as possible any differences likely to occur from changes in the amount of moisture after bottling.

Each chemist was also requested to carry out the method as ordinarily managed in his laboratory, working each sample with and without sodium chloride, side by side, and as nearly as possible under exactly the same conditions.

Twenty-two chemists took part in the work and their results were as follows: [See table inserted here.]

The method of Lindo-Gladding for potash determination is a very satisfactory one. The addition of NaCl is absolutely unnecessary, and will now be stricken out of the official methods.

The great differences in potash which often arise between guarantee of brand and anaylsis are not to be attributed to the method, but in most of the cases to the unequal mixing of the products.

I have shown in a paper read before the association at the August meeting of 1891, *Bulletin* No. 31, pp. 150, 152, the extreme difficulty of making mechanical mixtures sufficiently uniform to cover chemical guarantees, when for example eighty pounds or even 100 pounds of high grade muriate of potash at a guarantee of fifty per cent. potash, the bearer of the potash in a ton of mixture.

The same experience has been made by others, and the reports on potash give a vivid picture of this evil, which seems to grow as we go further South.

The report on determination of nitrogen was made by Mr. Van Slyke, of Seneca, N. Y. The report appears in the accompanying table. [See p. 7.]

The work on determination of nitrogen has, from the beginning, given the most satisfactory results.

The introduction of the Kjeldahl methods several years ago has given occasion for the careful testing of this method in its original and modified form.

In 1887, Prof. M. A. Scovell, of Louisville, Ky., introduced a modification to make the Kjeldahl method likewise applicable to nitrates by the use of salicylic acid (*Bulletin* No. 16, 1887, pp. 51–54) which at the meeting of 1888 was adopted as official.

The methods adopted as official methods:

(1) The absolute or cupric oxide method.
(2) The Kjeldahl method, not applicable in presence of nitrate.
(3) The Kjeldahl method, modified by Scovell, applicable to all fertilizers containing nitrates.

REPORT ON THE DETERMINATION OF NITROGEN (VAN SLYKE.)

I. Bone Meal.				II. Cotton-seed Meal.				State.	Analyst.	III. Sodium Nitrate.	
Kjeldahl Method with Permanganate.	Kjeldahl Method without Permanganate.	Gunning Method with Permanganate.	Gunning Method without Permanganate.	Kjeldahl Method with Permanganate.	Kjeldahl Method without Permanganate.	Gunning Method with Permanganate.	Gunning Method without Permanganate.			Ulsch Method.	Kjeldahl-Scoville Method.
first.	*second.*	*third.*	*fourth.*	*first.*	*second.*	*third.*	*fourth.*				
4·41	4·42	4·40	4·39	7·15	7·12	7·14	7·15	Arkansas.	Teller.		
4·46	4·48	4·50	4·42	7·17	7·23	7·31	7·03	Illinois.	Farrington.		
4·50	4·43	4·45	4·46	7·27	7·16	7·18	7·22	Kentucky.	Curtis.		
4·44	4·46	4·40	4·44	—	—	—	—	Michigan.	Miller.		
4·52	—	—	4·49	7·31	—	—	7·29	New Jersey.	Voorhees.	16·41	
4·47	—	—	4·45	7·30	—	—	7·21	New Jersey (Rutgers)	Cathcart.	16·33	
4·49	4·46	4·47	4·45	7·33	7·23	7·22	7·27	New York (Geneva)	Van Slyke.	—	
4·46	4·49	4·40	4·42	7·27	7·20	7·20	7·19	New York (Geneva)	Knisely.	16·17	
4·49	4·53	4·56	4·51	7·45	7·36	7·48	7·45	North Carolina.	Carpenter.	14·47	16·44
4·41	4·39	—	4·35	7·15	7·10	—	7·19	Pennsylvania.	Fields.	15·88	
4·52	4·38	—	4·56	7·34	7·04	—	—	U. S. Bio-Ch. Lab.	Emery.	—	
—	—	—	—	—	—	—	—	Department of Agricult.	Trescott.	16·34	16·36
4·47	4·49	4·48	4·47	7·32	7·24	7·29	7·26	West Virginia.	De Roode.		
4·42	4·38	4·45	4·31	7·24	7·14	7·28	7·05	Wisconsin.	Woll.		
4·47	4·43	4·46	4·44	7·27	7·18	7·26	7·31	Averages.			
4·52	4·53	4·56	4·56	7·45	7·36	7·48	7·45	Highest.			
4·41	4·39	4·40	4·31	7·15	7·04	7·14	7·05	Lowest.			
0·11	0·24	0·16	0·25	0·30	0·32	0·34	0·40	Difference.			

(4) The Ruffle method, applicable for all fertilizers containing nitrates.

(5) The soda-lime methods, not applicable for nitrates are all equally reliable in the hands of a careful analyst.

The Kjeldahl method is without any doubt an excellent method and especially adapted for station work, where the work can be suitably arranged to divide labor with a large quantity of samples of like character.

I find that most the station chemists work with this method, while on the other hand the trade chemists and most the chemists of manufacturers are using the soda-lime or Ruffle method, because, if one has to handle only a single sample at a time, one saves time in doing so.

Differences in low grade goods (two to three per cent. ammonia) should not be higher than 0·1 per cent., and in high grade never reach 0·5 per cent.

My experience during the last fifteen years has been that I never had material difference on this point with reliable trade or station chemists, but not so with chemists who instead of normal acid use diluted muriatic acid to absorb the ammonia and simply evaporate and weigh as muriate of ammonia. With such I never could agree.

It is hard to believe that at the present day such means should be applied by analytical chemists, but it has been the case until very recently, and is so doubtless yet in single instances.

On the whole the determination of nitrogen by the station chemists in the control of the work of fertilizers has given general satisfaction.

The most important part of a fertilizer material is without question phosphoric acid in its different forms, as water soluble, reverted or citrate of ammonia soluble, and insoluble, as well as total.

The determination of phosphoric acid is of such importance for the fertilizer trade that since the first meeting of the association strenuous efforts have been made to give concordant results.

How far the association has been successful is best shown by a recapitulation of the work done. I have gone back for five years, and now give you on accompanying tables the different reports in condensed form.

1888.

August 16, 17 and 18.

Three Samples Sent Out.

(*A*) South Carolina acid phosphate.
(*B*) English acid phosphate.
(*C*) Navassa acid phosphate laboratories have sent results.

	Moisture.	Soluble.	Reverted.	Insoluble.	Total.	Available.	Averages of Available Phosphoric Acid.
A	18·14	5·58	4·45	2·60	12·63	10·03	
	18·22	5·83	4·37	2·40	12·60	10·20	Mean, 10·45
	—	6·04	4·09	2·41	12·54	10·13	Highest, 11·51
	20·96	6·00	4·11	2·38	12·49	10·11	Lowest, 10·03
	17·25	6·10	4·65	2·35	13·10	10·75	
	18·10	6·15	5·36	2·17	13·68	11·51	Difference, 1·48
B	15·03	9·38	2·17	·43	11·98	11·55	
	17·07	9·48	2·24	·43	12·15	11·72	Mean, 11·31
	—	9·31	2·23	·20	11·74	11·54	Highest, 11·95
	16·15	9·34	2·41	·35	12·10	11·75	Lowest, 11·34
	15·65	9·35	2·00	·85	13·10	12·25	
	12·85	9·26	2·81	·50	12·57	12·07	Difference, 0·71
C	6·42	9·63	5·17	4·73	19·53	14·80	
	6·31	9·53	6·06	4·11	19·70	15·59	Mean, 15·31
	—	10·81	4·78	3·71	19·30	15·59	Highest, 15·99
	7·31	8·57	6·88	4·06	19·51	15·45	Lowest, 14·80
	5·40	8·60	6·10	4·25	19·25	15·00	
	7·45	8·78	6·68	4·32	19·78	15·96	Difference, 0·79

1889.

Five Samples Sent Out.

No. 1. Ground South Carolina phosphate.
No. 2. Ground tankage.
No. 3. Ammoniated super phosphate.
No. 4. Dissolved South Carolina phosphate.
No. 5. Dissolved Navassa phosphate.

AVERAGES.

	Number of Determinations.	Average.	Highest.	Lowest.	Difference.
Sample No. 1.					
Moisture,	17	.95	1.37	.76	.61
Phosphoric acid,	31	28.07	28.78	27.47	1.31
Sample No. 2.					
Moisture,	17	6.79	7.64	6.03	1.61
Phosphoric acid,	31	14.31	14.91	13.62	1.29
Sample No. 3.					
Moisture,	24	15.40	19.65	12.00	7.65
Soluble phosphoric acid,	32	7.11	8.57	6.00	2.57
Rev. " "	32	1.31	2.37	.80	1.47
Available " "	32	8.42	9.60	7.92	1.68
Insoluble " "	32	1.65	2.20	1.30	.90
Total " "	32	10.07	11.49	9.63	1.86
Sample No. 4.					
Moisture,	24	9.61	11.24	9.03	2.11
Soluble phosphoric acid,	32	10.88	11.51	8.67	2.84
Rev. " "	32	2.93	3.51	1.91	1.60
Available " "	32	13.81	14.29	12.37	1.92
Insoluble " "	32	1.95	2.81	1.28	1.53
Total " "	32	15.76	16.20	15.18	1.02
Sample No. 5.					
Moisture,	24	9.63	15.80	8.09	7.71
Soluble phosphoric acid,	32	7.52	10.04	6.00	4.04
Rev. " "	32	7.50	9.08	4.48	4.57
Available " "	32	15.11	15.90	14.52	1.38
Insoluble " "	32	3.40	3.86	2.93	.93
Total " "	32	18.51	19.19	18.06	1.13

1890.

Three Samples Sent Out.

No. 1. Guano.
No. 2. Acid phosphate.
No. 3. Ammoniated super phosphate.

AVERAGES.

	Number of Determinations.	Average.	Highest.	Lowest.	Difference.
Sample No. 1.					
Moisture,	8	13·91	16·02	12·05	3·97
Total phosphoric acid,	15	22·43	22·90	21·81	1·09
Sample No 2.					
Moisture,	10	16·35	17·82	15·20	2·62
Soluble phosphoric acid,	14	10·97	11·21	10·66	·55
Rev. " "	14	2·15	2·84	1·81	1·03
Available " "	15	13·15	13·64	12·64	1·00
Insoluble " "	15	1·81	2·16	1·53	·63
Total " "	15	14·96	15·48	14·38	1·10
Sample No. 3.					
Moisture,	8	10·99	13·65	10·18	3·47
Soluble phosphoric acid,	14	6·83	7·05	6·61	·44
Rev. " "	14	2·04	2·21	1·70	·51
Available " "	14	8·89	9·20	8·53	·67
Insoluble " "	14	1·76	2·02	1·57	·45
Total " "	14	10·67	11·04	10·15	·89

1891.

Three Samples Sent Out.

No. 1. A super phosphate.
No. 2. A complete fertilizer
No. 3. A complete fertilizer.

(Containing cotton-seed meal and other substances difficult to oxidize.)

AVERAGES.

	Number of Determinations.	Average.	Highest.	Lowest.	Difference.
Sample No. 1.					
Moisture,	16	6·21	9·65	5·06	4·59
Soluble phosphoric acid,	21	9·26	9·81	8·77	1·04
Insoluble " "	21	4·73	5·83	4·19	1·64
Total " "	21	13·54	18·30	16·96	1·34
Available " "	21	12·79	13·56	11·42	2·14
Sample No. 2.					
Moisture,	16	9·37	12·55	7·44	5·11
Soluble phosphoric acid,	22	6·46	6·87	5·80	1·07
Insoluble " "	22	·30	2·15	0·87	1·28
Total " "	22	11·40	12·12	10·96	1·16
Available " "	22	10·10	10 37	9·51	0·86
Sample No. 3.					
Moisture,	15	6·15	11·47	4·38	7·09
Soluble phosphoric acid,	22	6·42	7·07	5·83	1·24
Insoluble " "	23	2·94	4·37	2·32	2·05
Total " "	23	12·21	13·40	11·57	1·83
Available " "	23	9·27	9·92	7·20	2·72

1892.

Four Samples Sent Out.

No. 1. Super phosphates containing Keystone (iron and alumina) phosphate.
No. 2. Regular phosphate.
No. 3. Dissolved steamed bones.
No. 4. Thomas slag.

AVERAGES.

	Number of Determinations.	Average.	Highest.	Lowest.	Difference.
Sample No. 1.					
Moisture,	29	—	—	—	—
Soluble phosphoric acid,	21	1·92	2·39	1·67	0·72
Insoluble " "	21	7·92	9·21	7·23	1·98
Total " "	23	17·12	17·73	16·27	0·46
Available " "	21	9 20	10·23	7·56	2·77
Sample No. 2.					
Moisture,	22	—	—	—	—
Soluble phosphoric acid,	21	2·52	3·10	2·14	0·96
Insoluble " "	21	5·16	5·80	3·37	2·43
Total " "	23	13·95	14·43	13·15	1·28
Available " "	21	8·79	10·58	8·03	2·55
Sample No. 3.					
Moisture,	21	—	—	—	—
Soluble phosphoric acid,	21	2·39	2·61	1·75	0·86
Insoluble " "	21	8·01	11·25	6·72	4·43
Total " "	23	21·94	22·72	21·22	1·50
Available " "	21	13·03	16·95	11·47	5·48
Sample No. 4.					
Moisture,	17	—	—	—	—
Soluble phosphoric acid,	3	—	—	—	—
Insoluble " "	17	11·53	13·88	10·11	3·17
Total " "	17	18·40	19·14	17·69	1·45
Available " "	17	6·87	9·96	4·94	5·02

RECAPITULATION.

Difference in available phosphoric acid in the samples sent out by the association.

1888,	1·48	0·71	0·99	—
1889,	1·08	1·92	1·35	—
1890,	1·00	0·67	—	—
1891,	2·24	0·86	2·72	—
1892,	2·77	2·55	5·48	5·02

These figures speak for themselves and need no comment of mine. To say that they are satisfactory would be untrue. If you compare the results of the last five years, they apparently show a movement from bad to worse, but this is only apparently.

The samples of the first four years were mainly acid rocks, which are the easiest to handle, while the later reports are

composed of more complicated samples. This is especially the case with the samples of this year. [See full report of Professor Lord, table herewith.]

Sample No. 1 contained considerable Keystone phosphate, an iron alumina high grade phosphate, which has given rise to a great many controversies between chemists.

No. 2 is a regular mixture.

No. 3 is an incompletely dissolved bone, a material which always presents the greatest obstacles to proper filtration.

If you examine the results of No. 3 closely, with differences of over five per cent. available phosphoric acid, you might assume an extraordinary reason. We found in our laboratory that we had to quadruple the filtering paper before we could have a clear ammonia citrate solution, and are satisfied that the main reason for the extraordinary differences lay right there.

Sample No. 4, the Thomas slag, represents a new class of phosphates of lime, a quadribasic phosphate. It will take further study to ascertain if this class of products can be measured with the same rule as the bibasic phosphate in the common run of the fertilizers.

I have to call your attention to the great difference in the determination of moisture:

Sample No. 1.—Highest,	13·06
Lowest,	2·76
Difference,	10·30
Sample No. 2.—Highest,	20·13
Lowest,	9·40
Difference,	10 73
Sample No. 3.—Highest,	13·53
Lowest,	5·90
Difference,	8·63

The absolutely unsatisfactory results of the determination of moisture caused the association to discuss the reason for it at great length, and to the neglect of the main question of the shortcomings in the phosphoric acid determination. The brief time devoted to this year's report was

FULL REPORT OF PROFESSOR LORD, COLUMBUS, O., AS GIVEN AT THE LAST MEETING.

Analyst.	Sample No. 1.					Sample No. 2.					Sample No. 3.					Sample No. 4.				
	Moisture.	Water Soluble Acid.	Insoluble.	Total.	Citrate Ammonia Soluble.	Moisture.	Water Soluble.	Insoluble.	Total.	Available.	Moisture.	Water Soluble.	Insoluble.	Total.	Available.	Moisture.	Water Soluble.	Insoluble.	Total.	Citrate Ammonia Soluble.
K. P. McElroy,	7·12	2·06	7·51	16·86	9·35	13·99	2·73	4·35	13·87	9·52	6·90	2·44	4·86	21·81	16·95	0·05	—	8·54	18·50	9·96
A. W. Fries,	3·84	2·11	7·94	17·63	9·69	11·00	2·82	5·32	14·43	8·91	6·43	2·61	11·15	22·62	11·47	0·89	—	11·98	18·61	6·63
H. E. Curtis,	3·41	1·67	9·09	17·05	8·96	9·67	2·14	5·30	14·30	9·16	6·06	2·09	8·93	22·24	13·29	0·94	—	12·25	18·59	6·34
R. E. Noble,	3·89	1·70	7·41	17·12	9·71	10·69	2·41	5·39	14·40	9·01	8·08	2·26	9·50	22·44	12·94	1·46	0·07	10·27	18·54	8·27
S. Shiver,	3·78	—	—	17·24	—	10·05	—	—	13·91	—	—	—	—	21·58	—	0·99	—	—	18·17	—
R. N. Bracket,	—	—	—	17·21	—	—	—	—	13·93	—	9·07	2·56	10·00	22·00	12·10	1·20	—	13·28	18·20	4·92
J. S. Mong,	7·64	1·94	8·42	17·22	8·80	15·55	2·55	5·05	13·79	8·74	5·90	2·46	9·34	22·11	12·77	—	—	12·81	18·26	5·45
I. B. Carpenter,	9·76	1·93	8·13	17·19	9·06	9·43	2·63	5·26	13·78	8·52	7·42	2·52	8·80	21·49	12·69	0·83	—	12·44	18·17	5·73
R. C. Kedzie,	6·12	1·90	7·48	16·94	9·46	14·47	2·55	5·16	13·81	8·63	6·40	2·41	5·24	21·93	16·69	0·99	—	10·34	18·37	8·03
L. G. Patterson,	3·42	1·79	8·96	17·53	9·57	9·64	2·95	5·67	14·20	8·53	6·34	2·31	5·55	21·87	16·32	0·89	—	11·07	18·37	7·30
W. R. Perkins,	3·39	1·93	8·35	17·55	9·20	9·59	2·28	5·73	14·15	8·42	6·55	2·56	6·14	22·03	15·89	—	—	—	18·11	8·30
M. Farrington,	3·31	1·92	7·23	16·72	9·49	9·90	2·56	4·89	13·94	9·05	7·78	1·75	7·93	22·72	14·49	—	—	—	—	—
F. W. Morse,	3·36	1·74	6·81	16·27	9·46	10·76	2·31	4·08	13·15	9·07	6·55	2·07	6·82	20·92	14·10	0·47	0·03	9·07	17·69	7·72
J. L. Hills,	3·34	1·89	6·66	17·20	10·54	9·69	2·59	5·06	14·04	8·98	6·40	2·49	7·41	22·06	14·65	—	—	12·38	18·34	5·96
W. J. Davidson,	3·27	2·13	8·42	16·76	8·34	20·73	2·80	5·57	13·60	8·03	7·53	2·45	10·39	22·15	11·76	1·54	—	—	18·21	—
E. A. Heberly,	13·06	2·13	8·42	16·49	7·36	16·05	2·76	5·36	13·98	8·62	7·46	2·46	—	21·22	—	1·19	—	12·75	18·26	5·51
C. S. Cathcart,	8·73	1·93	8·03	16·87	10·23	16·05	2·42	3·37	13·95	10·58	7·73	2·23	8·14	21·31	13·17	1·21	—	—	18·32	—
F. J. Fulkenbach,	8·11	1·88	6·84	17·14	9·28	16·60	2·27	5·16	14·00	8·66	6·60	2·74	7·17	21·09	13·92	0·93	—	11·09	18·58	7·49
N. W. Lord,	3·47	1·68	7·86	17·14	9·28	10·80	2·27	4·79	13·93	9·14	7·60	2·30	8·69	21·79	13·10	1·22	—	13·10	18·47	5·37
W. B. Viets,	3·47	1·68	7·67	17·00	9·33	15·85	2·68	4·79	13·75	8·45	7·46	1·75	7·93	22·72	14·79	1·18	—	12·13	19·14	7·01
R. B. Moore,	3·80	1·92	7·80	17·42	8·21	14·92	3·10	5·30	14·32	9·02	—	—	—	—	—	—	—	—	—	—
Delaware River Chemical Works,	8·42	2·39	9·21	17·42	8·21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
F. Bargami,	—	—	—	—	—	10·00	2·56	5·63	14·00	8·37	6·53	2·16	10·36	22·40	12·04	0·75	0·26	10·43	18·82	8·39
W. J. Williams,	3·58	2·08	8·32	17·28	8·96	9·40	2·75	5·44	14·34	8·89	6·92	2·56	6·72	22·59	15·87	—	—	—	—	—
Average,	3·10	2·11	7·68	17·73	10·05	—	2·57	5·16	13·95	8·79	—	2·39	8·01	21·99	13·93	—	—	11·53	18·40	6·87
	—	1·92	7·92	17·12	9·20															

The column has available phosphoric acid added to it.

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absorbed without ventilating the serious irregularities in the results obtained on phosphoric acid.

What can be the reason for the utter failure of last year's researches on phosphoric acid?

The samples sent out by Professor Lord were prepared with the utmost care, which insured absolute uniformity. Should we assume that the balances have not been rightly adjusted? Professor Payne charges manufacturers with using bricks for weights. Have some of the gentlemen made proportionate mistakes in analysis?

The failure to reach closer results in moisture shows very plainly that the necessary care has not been taken in the work, and when in the simplest determination of all analytical work such blunders are possible, we need not wonder at the differences in other more complicated determinations. Analytical work will always bear the stamp of individuality, and no absolute identical results can be obtained, but the differences should not rise above a narrow limit.

The proposition, made at the 1890 session of the society, by Mr. Charles Gibson, of Chicago, to change the official method of phosphoric acid determination by making the ammonia citrate solution slightly alkaline (p. 170), caused me to have a series of researches made in my laboratory to establish to a certainty if such an addition of ammonia would work detrimentally to the interest of acid phosphates derived from lime phosphates.

The result of numerous analyses has clearly demonstrated that it is not possible to show any difference between the results obtained with a neutral solution of citrate of ammonia and a slightly ammoniacal solution.

As an example, I will give only two analyses made with our own acid phosphate:

	Neutral Solution of Citrate of Ammonia.		*Ammoniacal Solution of Citrate, Specific Gravity 1·90, added to 100 cc. Citrate of Ammonia.*	
	No. 1.	*No. 2.*	*No. 1.*	*No. 2.*
Total available phosphoric acid,	12·66	13·34	12·66	13·51
Phosphoric acid,	2·62	2·24	2·62	2·17
Total phosphoric acid,	15·28	15·66	15·28	15 68

Sample No. 1 is absolutely identical in its results.

Sample No. 2 shows only a difference of 0·07.

Being satisfied that in the change to a slight ammoniacal solution of citrate of ammonia from the strict neutral no direct harm will be done to the valuation of phosphates derived from phosphates of lime, our curiosity was aroused to see what acidity would show in the same material.

	Sample No. 1. 0·75 *Gramme Citric Acid to 100 of Citrate of Ammonia.*	*Sample No. 2.* 1·00 *Gramme Citric Acid to 100 of Citrate of Ammonia.*
Total available phosphoric acid, ...	12·85	13·70
Insoluble phosphoric acid, ...	2·43	1·98
Total phosphoric acid, ...	15·28	15·68
Increase of available phosphoric acid,	0·19	0·29

The increase of the available phosphoric acid under such strong acidity as is produced by the addition of 0·75 grm. and 1 grm. citric acid to 100 of the normal citrate of ammonia solution is only 0·19 and 0·29 per centum, a variation which is generally conceded to be permissible in analytical work, as shown by all previous reports of the society, proves to our satisfaction that the claim so often made, that the change of the ammonia citrate solution from neutral to acid causes great differences in the results of phosphoric acid determinations is not borne out by careful researches.

In order to test the stability of the citrate of ammonia solution, my assistant, Mr. Bergami, prepared, October, 1890, a bottle of normal citrate of ammonia solution, which has been kept in one-half litre bottles, glass stoppered, in a closet, and from time to time tested; the last test was made a few months ago, and proved the absolute neutrality of the solution.

In order to prevent the formation of fungus in the solution, we have, during the last few years, added to a quantity of about two litres, 0·1 gramme of salicylic acid. Without impairing the nature of the neutral solution in the least, this minimum quantity has successfully prevented the

formation of the fungus and thereby destroyed the possibility of changing the neutral condition to acidity.

This proves to our satisfaction that the claim of unreliability of the citrate of ammonia solution (p. 179, 180, Rep. of 1890) is not well founded, if all necessary precautions are taken in preparing the solution, and keeping the same in not too large bottles well stoppered, as recommended long ago by Prof. Paul Wagner, of Darmstadt.

The claim, made by the advocates of the ammoniacal citrate of ammonia solution, that the slightest acidity will impair the correctness of the analysis in alumina phosphates or producers, was almost incredible, and the publication of the analysis by Mr. Williams, pp. 178–179 of the last report, proves that the fault must lie somewhere else and not in the normal neutral solution of citrate of ammonia.

There are numerous other points in the determination of available phosphoric acid, which might let in small variations; temperature variations in the water-bath, the washing of the portion insoluble in citrate of ammonia, may influence the results, but I do not hesitate a moment to say that if the official methods were carefully executed by a competent analyst, such discrepancies would not be possible, even in case of iron and alumina phosphate. The Association of Official Agricultural Chemists has given us the very best methods for the investigation of fertilizers—methods that are accepted, with a single exception, over the whole United States (New Jersey alone determines available phosphoric acid at a temperature of 40° C., instead of 65° C., falling thereby one-half per cent. lower in average result). The association has now the imperative duty to establish the fact, that it only needs careful and competent workers to make such results as are laid before you to-night, absolutely impossible.

The fact that differences of two per cent. and six per cent. insoluble phosphoric acid are possible in test samples prepared with care for the very purpose of a trial of methods is calculated to upset all confidence in the so-called control work of the Agricultural Experiment Stations.

The difference in results as shown by the reports on phosphoric acid has very serious consequences for the fertilizer trade at large.

The stations have assumed the roll of arbitrators between buyer and seller. They have gone further, and act as advisers for the farmer by establishing valuations based on chemical analysis.

I will not ventilate to-night the real value of these valuations, but take facts as I find them.

The market valuation as adopted by the Eastern and Middle States is, I believe, seven and one-half or eight cents, per unit available phosphate. Supposing it is eight cents, and take sample No. 3.

The first analyst makes available phosphoric acid

$$\frac{16{\cdot}95 \text{ per cent.} \times 8}{135{\cdot}60 \times 20}$$

$27·12 valuation per 2,000 pounds.

The second analyst makes available phosphoric acid

$$\frac{11{\cdot}47 \times 8}{91{\cdot}76 \times 20}$$

$18.35 per ton.

Or difference in valuation of $8.67 per ton. On which analysis shall we buy? is a question which readily will come to the farmer.

Supposing Sample No. 3 represents a brand guaranteed thirteen per cent. of available phosphoric acid.

In several Southern States a falling off of ten per cent. of the guaranteed analysis makes the goods liable to confiscation.

In this case two analyses of State chemists, the second and sixteenth, will brand the goods as to be condemned under the law. But if the manufacturer should have been misled by one of the numerous high results, and put his guarantee high, he made himself a criminal in half the States of which you have the returns before you.

As long as the results of chemical analysis are so uncertain, as shown by the work of the official association

under the most favorable examination, the public should be extremely careful in forming judgment on a single report of a station if the same should show a remarkable difference between guaranty and results.

The fertilizer trade has been maltreated by suspicion, perhaps, more than any other trade. The trade has been surrounded by an endless chain of different State laws; some of which are in conflict with the Constitution of the United States, and these laws are, to a great extent, based on the idea of the infallibility of chemical control. How infallible the States' control can be, I have shown you to-night by the work presented.

The work of the Association of Agricultural Chemists deserves to be highly commended for the good it has done, but the association must go further, before it can hope to be recognized as the arbitrator in the fertilizer trade.

The association should influence State legislation through its members, to the end that the laws regulating the trade shall be made to conform with the practical results of their work.

The influence of the association should be exerted likewise in the legislation of New Jersey, to effect the repeal of an antiquated dictum in analytical work erroneously sanctioned by law; and in the Southern States, where unjust laws have been based on the belief in the infallibility of control work of the agricultural station.

DISCUSSION.

DR. WM. J. WILLIAMS.—I have listened with much pleasure to Dr. Terne's paper on the discrepancies found in the analyses of different official chemists when engaged in analyzing the same goods, and not merely the same goods, but actually the same (divided) sample of the same goods.

Dr. Terne has called attention more particularly to the analyses of bone—raw and partially dissolved. Similar cases have arisen in my experience, with even greater discrepancies in dealing with our Keystone concentrated phosphate, a concentrated phosphate of iron and alumina. While several chemists of high standing in the profession,

both official and non-official chemists, gave the quantity of available phosphoric acid fairly closely, as, for instance:

Per Cent.	*Per Cent.*
30·60	30·25
30·87	30·35
31·02	30·37
32·35	

others could only get 25·22 per cent. and others 20·31, 19·40 and 19·70 per cent. The last three especially show a difference beyond any reasonable limit.

Now, as the verdict of an "official" chemist is final in his State, or at any rate his figures are published and become generally known, such discrepancies work great injustice to manufacturers, and if the official chemists could make some change in the "official" method, whereby these discrepancies could be minimized, it would be a great advantage to all concerned.

The question has already been discussed by the official chemists in convention, but up to this time no solution of the difficulty has been arrived at.

One point suggested was that a slight acidity or alkalinity of the "official" neutral ammonium citrate solution caused the greater part of these differences. So far as my own investigations go (made since the convention above referred to), I do *not* find that a *slight* acidity or alkalinity of the solution makes any appreciable difference greater than the ordinary errors of analysis.

I find, however, that a slight difference in the time of exposure of the material at 65° C. does make considerable differences in the results. The instructions now given read: "Cork the flask securely and place it in a water-bath, the water of which stands at 65° C. * * * Raising the temperature as rapidly as practicable to 65° C., which is subsequently maintained, digest for *thirty* minutes *from the instant of insertion*," etc. Up to 1887, I think, or possibly 1888, the instructions were to this effect: Place the flask in a water-bath, the temperature of the water in which is 65° C., raise the contents of the flask up to 65° C. as rapidly as possible, and *maintain at that temperature* for thirty minutes. (I cannot

give at the moment the exact words, but that is the substance.)

Now, it would appear that this difference of instructions should not be of much moment, as all using the official method are working under the same instructions, and in all cases the time of exposure to a temperature of 65° C. is of equal duration, but such is not actually the case. Under the earlier instructions the contents of the flask were uniformily exposed to a temperature of 65° C. for thirty minutes; and under these conditions these great discrepancies in results appeared neither so frequently nor to so great an extent. But when the instructions were changed to the present form, viz: "Digest for thirty minutes from the instant of insertion," then they became greater and more frequent.

This, I think, is due to the differences in the conditions to which the material is exposed in the water-bath; that is to say, the solution of exposed neutral ammonium citrate reaches 65° C. much quicker in some cases than others. I made inquiries from many chemists on this point, *i. e.*, how long it required for the solution in the flasks to reach the temperature of 65° C. after insertion in the water-bath. The time varied from three to nine or ten minutes. That is to say, the time during which the material was exposed to a temperature of 65° C. varied from twenty to twenty-seven minutes, a variation which is, I believe, sufficient to account for the greater part of the discrepancies. This variation in time can be ascribed to many causes, *e. g.*, size of bath, depth of water, quantity of heat used, thickness or shape of flasks or other vessel, etc., and it would appear impossible to produce uniformity of conditions, and, therefore, uniformity of results under the present directions. I think, however, if the instructions were modified, so as to secure the exposure of the material to a temperature of 65° C. for a uniform fixed time of thirty minutes, as was done under the original form of instructions, instead of to a variable time ranging from twenty to twenty-seven minutes, as under the present form of instructions, that the results would be far more uniform and would be far more satisfactory to all concerned.

OBSERVATIONS ON FERRO-TUNGSTEN.

By Wm H. Wahl.

[*Read at the stated meeting of the Chemical Section, held November 15, 1892.*]

In the course of an investigation carried on during the past two years by the writer, in conjunction with Dr. William H. Greene, with the object in view of producing pure ferro-alloys, we made a number of experiments with ferro-tungsten.

These tungsten alloys exhibited similar physical properties possessing considerable hardness and toughness, an extremely fine crystalline texture, with a fracture resembling that of tool-steel, and a specific gravity ranging between 9·3 and 10·14.

Some of the fractures, however, exhibited the fact that the alloy was not entirely homogeneous, disclosing under the glass and in places to the eye, the presence of what are apparently smooth cleavages of imperfect crystals scattered through the finely-crystalline matrix of the alloy. The specimen shown you will illustrate my statement.

A sample of the alloy of 10·14 specific gravity was analyzed at my suggestion, by Mr. J. F. de Benneville in the laboratory of Dr. Genth. The result of the analysis exhibited a very high percentage of tungsten in the sample, and its behavior towards liquid and fused solvents proved the interesting fact that a large proportion of the tungsten was present in the uncombined condition, as metallic tungsten, crystallized in the matrix of the alloy. The facts upon which Mr. de Benneville has founded this observation will appear in the following extract from a letter describing briefly the method pursued in his analytical work.

"*Aqua regia* attacked it, although not energetically, and by decanting and adding fresh portions of acid from time to time a residue was obtained which resisted further action by acids, or by fusion with Na_2CO_3 and KNO_3. It was a heavy, black, pulverulent substance, and in its negative

action towards solvents answered to tungsten, which I took it to be. It gave 22·54 per cent. of the original material. A second portion of the material (powdered in a steel mortar) was fused with a mixture of Na_2CO_3 and KNO_3, lixiviated and the residue weighed. This was fused again and the residue weighed, the second weighing being practically the same, and yielding 22·80 per cent. of the original material, using strong HCl (1·20 Sp. gr.) in successive portions, decanting, igniting the residue and treating again with acid, gave me 21·74 per cent."

The foregoing extract appears fully to justify the conclusion that the undissolved residue represents the metallic tungsten present in the alloy in the uncombined state.

The composition of the metal analyzed by Mr. de Benneville is as follows:

	Per Cent.
C,	0·85
P,	0·041
Si,	0·14
Mn,	trace
Fe,	42·28
W (metal),	22·54
W (alloy),	34·35
	100·201

A study of these figures reveals another interesting fact to which I desire to call your attention.

Taking the figures of the iron and combined tungsten (42·28 : 34·35) and calculating the percentage of tungsten which this ratio represents, we obtain 44·82 per cent. tungsten. A calculation shows also that the figures representing the compound Fe_4W are almost identical with the ratio above-named, to-wit:

Fe_4W Found.	*Fe_4W By Theory.*	*Difference.*
Fe, 55·18 per cent.	Fe, 54·91 per cent.	
W, 44·82 "	W, 45·09 "	— 0·27
100·00	100·00	

The conclusion would seem to be justified by the facts above noted, that the saturation point of iron for tungsten

is represented by the ratio exhibited in the compound Fe_4W, and that any excess of tungsten, above this ratio, present in a tungsten iron will remain uncombined.

I wish to express this statement as *probably* the correct interpretation of the facts, for it is hazardous to make a generalization of this kind on the results of a single analysis.

It may be interesting to note in conclusion that Howe, in his "Metallurgy of Steel," refers to several cases of ferro-tungsten indicating the composition Fe_8W, and that the works on metallurgy as a rule accept without question the dictum that iron and tungsten will unite in all proportions. In the light of the facts given in this paper, this last statement requires qualification.

PROCEEDINGS

OF THE

ELECTRICAL SECTION

OF THE

FRANKLIN INSTITUTE.

[*Stated meeting, held Tuesday, October 25, 1892.*]

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, October 25, 1892.

Prof. Edwin J. Houston, President, in the chair.

Present, thirty-four members and visitors.

The minutes of the previous meeting were read and approved.

The Treasurer reported the cash balance in the treasury, and presented bills for printing and clerical work, which were approved and ordered paid.

The Committee on Admissions reported ten elections to membership since last meeting. Two nominations were referred to this committee.

The first sentence of Article IV of the Section's By-Laws was amended to read, "The annual dues of active, corresponding and associate members shall be two dollars, payable annually in advance, on the first of January of each year."

Mr. Elmer G. Willyoung exhibited "A New Ballistic Galvanometer," and read a paper describing it. The instrument is made by Queen & Co., its principal features being the composite coils consisting of about five different sizes of wire forming portions of the coils, whose shapes are obtained by careful calculation for best effect; the magnetic system consisting of four bell magnets, the lowest one carrying a movable ring, by means of which the sensibility of the system can be varied without using a controlling magnet; and the quartz fibre suspension. The paper was referred for publication.

Mr. Paul A. Winand read a paper "On the Measurement of Energy in the Three-Phase System," in which he described the best methods of obtaining correct results. Referred for publication.

A communication was read from Mr. E. G. Willyoung inviting the Section to visit the Electrical Laboratory of Queen & Co., at Ardmore, Pa., and upon motion the chair appointed Messrs. E. A. Partridge and Rondinella a committee to make the necessary arrangements.

The meeting then adjourned.

L. F. RONDINELLA, *Secretary.*

A NEW BALLISTIC GALVANOMETER.

(*a*) WITH HIGH INSULATION, GRADED COILS, AND OTHERWISE CONSTRUCTED CLOSELY TO THEORY.

(*b*) WITH A NEW METHOD OF VARYING THE SENSIBILITY WITHIN THE WIDEST LIMITS WITHOUT THE USE OF A CONTROL MAGNET.

By Elmer G. Willyoung.

[*Read at the stated meeting of the Electrical Section, Tuesday, October 25, 1892.*]

(*a*) The galvanometer which I am about to present to your notice is not, as are many instruments, the result of chance development, but represents the fulfilment of deliberate intention aided by close application of theory and experimental data systematically obtained for the purpose. Some six or eight months ago it became necessary for the firm with which I am connected to place upon the market a good ballistic galvanometer, and I accordingly set about to design and build such an instrument. After a little time spent in looking over a number of combinations and general styles advocated by various persons, and in pondering different theoretical considerations, I designed an instrument somewhat similar to that which you see. (*Fig. 1.*) This first instrument, when completed, was taken in hand at the laboratory and thoroughly tested under a large variety of conditions; its law of deflection, its leakage, the decrements of damping with coil frames open and closed; with coils short-circuited, and in series with various resistances, were all found.

Various modes of suspension and of constructing systems were also tried. In particular, a considerable time was devoted to a study of different methods of varying the sensibility of the system and some important conclusions were drawn. After this earlier and systematic study of the instrument it was placed in the laboratory, and used for the

regular daily work of determining battery curves, condenser capacities, making hysteresis determinations, etc., which naturally make up the routine of a working laboratory. After some weeks of work of this kind and accompanying "unconscious cerebration" of the writer, and of one of his associates, Mr. Northrup, who made most of the systematic tests first referred to, the matter was again taken up, a modified design produced, and the instrument constructed which you see here. I will first refer to more general points which are to be noticed.

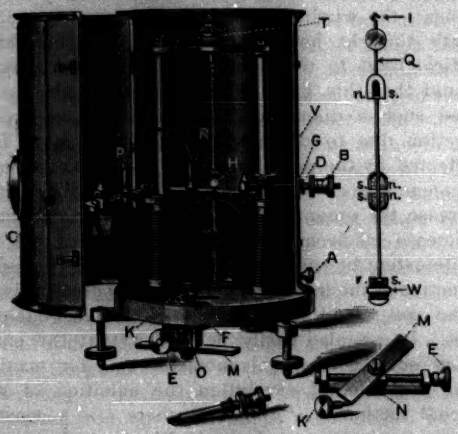

FIG. 1.

To speak of the general plan of the instrument for a moment, the coils are two in number and equal in size; they close upon the magnetic system, which hangs in a diameter between them, and are supported upon highly polished and corrugated hard rubber pillars. These pillars are two and seven-sixteenths inches high, and, since the corrugations are sides of equilateral triangles, they are equivalent to plain pillars of just twice the surface or about five inches high. The surface resistance to leakage is, therefore, very great. Upon the rubber pillars are brass rods joined at the top by

a cross bar, which carries the torsion head and its suspension. The suspension is, when in position, a trifle over two and three-fourths inches long. The torsion head is divided into 15° divisions, and by its means the coëfficient of torsion of any suspension may be quickly obtained. I want to call particular attention to the coils of this galvanometer; in this regard the instrument has been as closely conformed to theory as possible. It is, of course, a fundamental fact of galvanometer theory that the electro-magnetic effect of a coil upon a magnet is directly proportional to the number of turns in the coil, and hence, in a very sensitive instrument, the windings must be as many as possible; a moment's thought, however, makes it clear that there is a practical limit to the number of turns which may be used, and that this limit is reached when the coil has obtained such a diameter that the loss of electro-magnetic action due to increased resistance is equal to the gain effected by the extra turn. In this we are assuming that but one size of wire is used in the winding; if now we increase the cross-section of the wire suitably we may still increase efficiency by adding turns, and this to an indefinite extent by continuing to increase the cross-section as a function of the increased diameter; by doing this, however, the coil would soon become of unmanageable proportions. It can also easily be shown that the exterior bounding surface of a galvanometer coil, for maximum effect, is a surface of revolution the equation of whose meridional section is $u^2 = c^2 \sin \theta$ where c is a constant.* The form of this curve is shown in *Fig. 2*, which represents a meridional cross-section of a properly-shaped coil, where there are several curves based on different values of c. It is obviously not commercially possible to vary the wire as continuously as rigid conformity to theory would require, as this would mean a continuous wire tapering uniformly from one end to the other; instead of this we may, however, obtain an approximation to theory by changing the sizes of wire used a greater or less number of times. This is what

* Maxwell's *Elec. and Magnetism*, Vol. ii, p. 361 (3d ed.).

has been done in the coils before you,* and, indeed, is what is being done now in all galvanometers made by our firm. The usual number of changes which we make is five, and that number has been employed here. Galvanometer coils wound in this way are called "graded" coils. In effecting this grading we first decide upon the total resistance of the coil and then by an application of the mathematical theory, as given in Maxwell, determine the exact diameters of the wires which must be used and the positions and shapes of their bounding surfaces, the latter, of course, satisfying the equation $w^2 = c^2 \sin \theta$. The labor involved in making these calculations is considerable, as the insulation thickness of the wire is a complicating factor which must be allowed for; it is, in fact, necessary to obtain the final result by a series of approximations of at least three in number, first substituting an approximate value of δ, the ratio of insulation thickness to the radius of the bare wire, from which we obtain an approximate value of this bare wire radius; we then substitute a more correct value of δ and solve again and at last use the nearest gauge number to the obtained diameter. We do not, of course, work all these calculations out every time we wind a pair of coils, as that would not be commercially possible. Our coils on all our galvanometers are of the same size, or rather are reduced to several standard sizes and gauges are used for each size. Consequently we have been enabled to work out the complete winding for each size of coil for every resistance likely to be desired from one-half ohm to 10,000 ohms; we have a typical drawing, properly lettered, and all that is done is to wind wire until the bounding curve of the wire surface calipers to the determined data.† I have here (*Fig. 3*) one of the forms upon which the coils are wound. We have a number of these for each of the two or three types of coils, all of each type being rigidly the same size. They are made so that after a coil is wound the sides and core can readily

* A specimen coil, wound and ready to slip into any galvanometer frame, was here shown.

† This typical drawing was here shown together with the table of calculated data accompanying it from which coils are actually wound.

be slipped away. The core is of steel, in order that it may be very small and yet be of sufficient strength. Note the peculiar curve (*A*, *Fig. 3*) at one side; this is a mathematically determined curve, and is so shaped as to secure the maximum action upon the needle of the windings lying close to the magnets. Here is a wound coil ready for slipping into the frame; I think you can all see the lines of separation of the various windings, and that they have a greater diameter upon the one side than upon the other. In the pair of coils used in this instrument the space surrounding the magnets has been made very small, so as to make use of the exceedingly valuable space lying close to the centre. To show the practical advantage of grading coils, if the coils of this

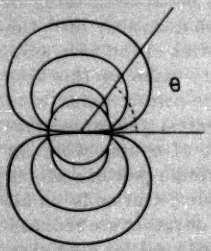

FIG. 2.

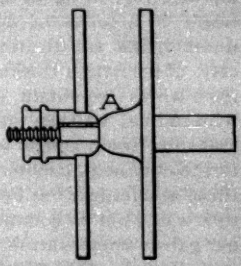

FIG. 3.

galvanometer had been wound to the same resistance with but *one* size of wire, and occupying the same volume, the sensibility would have been but four-ninths as much. The difference in cost is very inconsiderable.

The clearance between the coils in this instrument is but one-twentieth of an inch, leaving only just enough room for the shaft of the system to work freely. The coils may be used in any way desired, either singly or in series or in multiple, thus giving one three instruments in one, *e. g.*, the present instrument is of about 5,000 ohms resistance with the coils in series; using either coil singly the resistance is 2,500 ohms and in multiple 1,250 ohms. The various connections are quickly made by means of the

little plug tipped flexible cords (*R*, *Fig. 1*), which plug into little receptacles *P* on the coil frames. In case a greater range even than this is desired, the coils are made interchangeable and may readily be removed from the instrument by simply loosening one screw on each frame. The binding posts *B* are placed at the extremities of brass rods which pass through the case and screw upon lugs attached to the coil frames; where the rods pass through the case, the case holes *G* are bored somewhat larger and a rubber collar *D* fits upon the rod and fills up the holes in the case, keeping out dust and dirt; when the instrument is to be used the collars are drawn back so that there is no possibility of leakage taking place through the case, and the entire instrument is still supported by the corrugated rubber pillars.

It will be noticed that there are no levels upon this instrument; they are left off purposely. I don't know that I had ever seen a good galvanometer without levels attached until I began to leave them off of our instruments. I had never questioned the utility of them while I was simply a user of apparatus, although I never made any use of them myself, but when it became a matter of business and my attention had to be given to the subject of designing the best for the lowest cost, I soon came to look upon the level as an utterly superfluous appendage. Theoretically, of course, the level has been upon the instruments for the purpose of adjusting them *to* the system, so that when the bubble was in its proper position the system would hang perfectly free; practically, however, the clearance allowed in a good galvanometer is so small that the slightest lack of perfection in the adjustment or change of position of the torsion head would cause the system to stick; the smallest hair or shred of insulating material or stray fibre would operate in the same way, so that actually the user has always found it far more convenient to level the instrument up by means of the levelling screws, until, by observing the mirror, the system was seen to swing free. If this was not easily attained the coils would be swung apart, and the system and faces of the coils closely inspected to ascertain the

cause of difficulty. I have inquired of many experienced users of these delicate galvanometers, and have always received the same reply, viz: that they never thought of using the levels and found them entirely unnecessary, thus confirming my own experience. In order quickly to inspect the system, the coils are pivoted so as to be easily thrown open by merely removing the binding post rods and raising

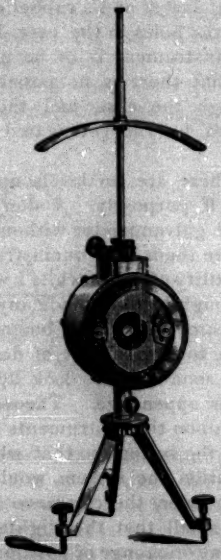

FIG. 4.

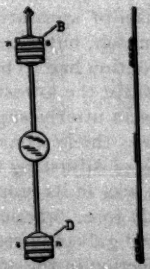

FIG. 5.

the milled screw clamp *V*. In practice the first coil only is swung away.

(*b*) Having thus described the more general characteristics of the instrument, I want to speak more particularly of an improvement in galvanometers having moving systems composed of magnets which has been devised in our laboratory and applied for the first time in this form of instru-

ment. The improvement consists in a means of varying the sensibility of the instrument within the widest limits in an exceedingly simple way, and in a manner which entirely does away with some of the annoying inconveniences and sources of inaccuracy which have been experienced heretofore when using instruments of this character. The usual method of varying the sensibility of galvanometers of this kind has been, as is well known, by neutralizing to a greater or less degree, the influence of the earth's field upon the system by means of a control magnet suitably placed upon the instrument. This magnet has usually been placed upon a standard attached to the instrument and rising from its centre above as in the galvanometer shown in *Fig. 4*. Occasionally this magnet has been placed below instead of above, and in one or two patterns at the front or back. This magnet could be raised or lowered to change the intensity of its field at the needle, or oriented, in order to change its plane; occasionally, also, a means of changing the intensity of the control or its plane without moving it to or away from the needle has been employed.* All these plans have been somewhat expensive to use, and all, except the last, have been open to the objection that they could not be used without jarring and disturbing the instrument. But the most serious difficulty has been due to an entirely different cause, which I will endeavor to point out. Let us conceive of the case of a system composed of two sets of magnets, *B* and *D*, *Fig. 5*, the upper set composed of four magnets with their north poles pointing in one direction, and the lower set composed of three magnets of the same length as the upper set, and with their north poles pointing in the opposite direction; this combination is known as an "astatic" system, since the controlling force of the earth's field is only the *difference* of the magnetic moments of the two systems. In the case supposed, the upper set "controls," and the north poles of the upper system will point to the north. If now we wish to make the system still more astatic we weaken the earth's field upon the upper needle by means of

* See Kittler's *Handbuch der Elektrotechnik*, pp. 226, 227, 228, Vol. i, 1886.

a control magnet suitably placed near it.* We now have the magnetic system subject to the influence of two fields, one the earth's and the other the field of the control magnet. We may represent these two fields in strength and direction by lines suitably drawn; let us also suppose that these fields are placed to oppose one another, but with planes not rigidly parallel. *Fig. 6* (*a*) shows this with *A B* representing the earth's field in strength and direction, and *A C* that of the control magnet. Completing the parallelogram we obtain *A D*, which represents the resultant field in direction and intensity. Let us suppose, now, that the earth's field slightly changes in direction, as is shown in (*b*) *Fig. 6*. The resultant now point, nearly directly opposite to its first position, and though the earth's field has moved through but an exceedingly small angle, we see that the resultant

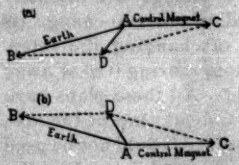

FIG. 6.

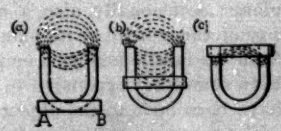

FIG. 7.

will have moved through nearly 180°, so that the slightest shifting of the earth's meridian causes an immensely magnified movement of the resultant. This theoretical conclusion is thoroughly realized in practice, although but few who are annoyed by it are aware of its cause. It is, of course, seen that this magnification varies with the sensibility and is largest when the instrument is most sensitive. In galvanometer work this phenomenon is known as the "drift" of the needles, and the tyro in galvanometer practice is very apt to think his instrument bewitched for some time after

* This is the method which has always been employed in instruments which have been used; it is, however, not theoretically the best way, which would be to place a standard at right angles to the earth's meridian and midway between the sets of magnets, and place upon this standard a control magnet, rotating upon it in a plane at right angles to the standard; its horizontal component would vary with its phase.

making its acquaintance. There are none of us, probably but who have experienced it; we work at an instrument, adjust the telescope and get the mirror nicely on zero. In a few minutes we look again and find some other number of the scale before us, and if we watch for a time, we may see the scale travelling along the mirror, until often it disappears entirely, the mirror having entirely left it; if we let it alone it will very likely come back later in the day and go in the other direction. The mirror is never still, going now one way and now another, though more stable during some parts of the day than at others. This is apt to be a very serious hindrance in making exact observations when the instrument is very sensitive, as when the mirror fails to return to zero after a deflection we are unable to tell definitely whether this is due to the cause just mentioned or to some other, and, if the former, as to just how much must be added to or subtracted from the observed deflection on this account. It also necessitates frequent re-adjustment and hence, re-determination of the constant of the instrument to prevent the mirror going entirely off the scale. I will now explain the improvement embodied in this instrument, and now for the first time made public. The device is the invention of Mr. E. F. Northrup, of Queen & Co.'s laboratory, and I regard it as one of the most important additions to galvanometer practice which has been made for many years. By its use all the annoyances of "drift" are eliminated and the instrument rendered much more certain in its indications besides being less costly to manufacture. Reference to *Fig.* 7 will show the principle of the device; *M* is an ordinary horseshoe magnet at the bend of which is a bar of soft iron *A B*; the distribution of the lines of force is practically unaltered by this bar. In (*b*) we have the same magnet, but with the bar moved about half-way up towards the poles; the lines of force are now considerably deformed, and many have been drawn down so as to pass through the soft iron armature. In (*c*) the armature has been pushed up to the very poles, and almost all the lines are now passing through it, the magnet thus being short-circuited and made about as ineffective, as regards its exter-

nal field, as would be a mass of soft iron of the same shape. Now, let us look at *Fig. 8*, which represents the system used in this instrument. The system is seen to be composed of four "bell" magnets of the well-known Siemens type; the upper and lower have their poles turned in the same direction while the centre magnets have their poles also in the same direction but opposite to that of the outside magnets. The upper and lower magnets thus make up one set of an ordinary astatic system, and correspond with

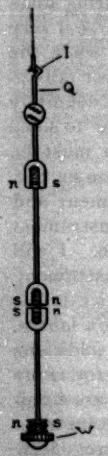

FIG. 8.

FIG. 9.

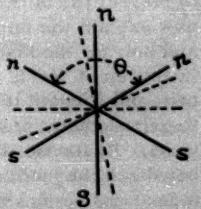

FIG. 10.

the set *B*, *Fig. 5*, while the centre magnets correspond with the set *D*. The outside magnets are, as will be noticed, slightly longer than the inner ones, and will, consequently, if all are magnetized in practically the same way, be the stronger, and will "control" the system. If now we weaken one or both of these outside magnets their control will be lessened, and the period of the system will be increased, until finally the influence of the inner magnets will begin to predominate, and the system will swing round through 180°. The means by which this weakening

is accomplished are in this instrument very simple. The lower magnet has cut upon it a very fine screw thread, and upon this thread is fitted a narrow and light ring of soft iron *W*. When the ring is at the bottom of the magnet its influence is *nil*; if we desire a greater sensibility we simply screw the ring up until this sensibility is attained. In order to do this easily, the system is attached to the fibre by means of an exceedingly small hook *I*, and may readily be removed by simply lifting it off with the aid of a small pair of forceps when the ring may be moved a trifle, the system replaced and the vibration period observed. In practice all this can be done in a very few minutes. It might be objected that in thus "short-circuiting" this outside magnet we lose a portion of the deflecting moment of this magnet, which is, of course, acted upon by the coil. This is true but the loss is very small, the effective couple of these outside magnets being vastly less than that of the inner. The best proof, however, is the result, and here are some of the results obtained with this instrument. Using the old method of control, *i. e.*, an exterior control magnet, we had:

Period (single swing),	$4\frac{1}{2}\frac{7}{8}$ seconds.
Mean deflection $\frac{1}{8}$ M. F. at 1·44 volts,	100 mms.
Drift in 5 minutes,	2·5 mms.

With Northrup's method:

Period (single swing),	$11\frac{1}{10}$ seconds
Mean deflection $\frac{1}{8}$ M. F. at 1·44 volts,	21·5 mms.
Drift in 5 minutes,	Inappreciable.

And making still more astatic:

Period (single swing),	21·6 seconds.
Mean deflection $\frac{1}{10}$ M. F. at 1·44 volts,	237 mms.
Drift in 5 minutes,	Inappreciable.

The large vibration period of over forty seconds for a complete swing obtained without "drift" is extraordinary in my experience; it may readily be made much greater, however, and the instrument correspondingly more sensitive if desired by this method. I have here a system made in the same way as the one in the instrument before you, but larger, so that I may illustrate the working of this

device. It is hung by a silk fibre with the short-circuiting ring at the bottom, and the system is as insensitive as it can be made with the outside magnets controlling. I start it vibrating, and we note that the period is very short, something like three seconds (*s. v.*). I now screw up the ring about four turns and you will notice that the period is considerably lengthened and is now about ten seconds (*s. v*). Now, it is about twenty seconds (*s. v.*), and by screwing it up further I can completely reverse the plane of the system; if I continue to still further weaken the lower magnet I shall obviously make the system less and less sensitive, since the centre magnets will control more and more strongly. Instead of screwing this ring on we may, of course, merely slip it on, allowing it to hold by its own friction only. We may also apply it to the upper instead of the lower magnet, or to both if we desire. The application of this same method to the orthodox light system is obvious and may be made in a variety of ways. *Fig. 5* represents in diagram such a system as used in the Thomson four-coil galvanometer as made by us, and *Fig. 9* the system as it might be modified to use Northrup's device. Here there are two sets of magnets, *B* and *D*, which would lie in the centre of the upper and lower pair of coils, respectively.

To use the new device we make the shaft of the system long enough to extend down a little below the bottom of the lower coil and place upon it an extra magnet *C*, as shown in the figure, with its poles placed in the same direction as those of the upper set. At the back of this little extra magnet we pivot a small piece of soft iron *Z*, of about the same size and shape as the magnet, so that it may be rotated to be either perpendicular or parallel to it. When in the former position, the little extra magnet is allowed to assist the upper set which consequently controls; when in the latter, the extra magnet is short-circuited, and the influence of the lower set of magnets, being a trifle longer, predominates. The weight added to the system by this construction may be so small as to be inappreciable in all except the most delicate instruments. We ourselves, however, have not yet adopted this device as applied to very light and small

systems, on account of the excessive delicacy of construction involved and of lack of time in which to work this out with thoroughness. The necessary delicacy of such a "light" system is so very great,* mechanically, that I have doubts as to whether the device had best be applied to such. In the instrument before you, I use an exterior control magnet when the light system is used. This magnet you see in position upon the base (*Fig. 1*); a middle head *E* works a screw through a block *O*, so that the position of the magnet with reference to the system is easily varied. When the ballistic system is used, this magnet is easily removed by unscrewing the small head *K* upon the upper part of the base.

The general construction of the system used in this instrument is well worth looking at, as considerable pains have been taken to work out one thoroughly adapted to the requirements. As is well known, a perfect ballistic system must have so large a moment of inertia that the entire quantity of electricity which it is designed to measure must have time to pass through the coils before the system is able to move. In addition it must be so shaped as to have a minimum damping either of air friction or magnetic or otherwise. To avoid air friction, the system must be of small diameter and shaped so as not to scoop and carry air before it and it must be very heavy in order to give it the required large moment of inertia. This system, I think, thoroughly conforms to all the requirements. The magnets are cylindrical bell-shaped magnets of but thirteen–sixty-fourths inches in diameter and are threaded upon a shaft of quartz rod; in order to make them very heavy they are filled with small fragments of lead, held in place by means of a little hard wax; after being filled thus a little brass washer is placed in the end of each magnet with the shaft passing directly through its centre; this is for the purpose of holding the magnets in place in a perfectly symmetrical manner. The whole weight of this system is 4·5 grammes. It is

* A light system, as made by us for Thomson reflecting galvanometers, weighs somewhat less than 350 milligrammes.

made with a small hook, *I*, above, so that it can be quickly unhooked from the fibre for adjustment or to be replaced by another kind of system. The fibre also hooks into place and can be quickly removed in correspondence with the system which is to be used. For the most delicate work the use of hooks introduces a slight error as the two hooks will "ride" upon one another through a very small angle and thus the system will deflect without torsion of the suspension. In such cases a small drop of hot beeswax touched to the hooks will cement them together in a perfectly satisfactory manner, and they are readily separable again by touching for an instant with a warm iron or brass wire.

The suspension itself of quartz and is about $\frac{1}{10000}$ of an inch in diameter; quartz is used because it has absolutely no set whatever, a statement which is true of no other suspension, metallic or otherwise, of which I have knowledge, its elasticity is almost perfect through extremely large limits. According to Professor Threlfall a quartz fibre will stand one-third of a turn for every one-half inch in a fibre $\frac{1}{10000}$ inch in diameter. It is also unaffected by ordinary atmospheric or temperature changes, and has so great a tensile strength that, for the same system, its coëfficient of torsion will be less than that of the finest silk fibre. We are able now to draw these fibres out to any length and to any degree of fineness with great facility. You may wonder how the hooks are placed upon the fibre: by simply platinizing the ends and soldering the hooks to them. When we first established our laboratory at Ardmore, all the suspensions of all our galvanometers on this general type were of silk; now I doubt whether a silk fibre can be found anywhere, all are either quartz or metallic. Some of these fibres have been made and are occasionally used for the most delicate suspensions, so fine that it has been impossible to see them except by the interference colors which are produced; to the eye there is a flash of beautiful iridescence, nothing more. Their tensile strength is very great, over 150,000 pounds to the square inch in the finer sizes. I made mention of the fact that the magnets of the system are threaded upon a quartz rod; there is a reason for this also, which we have

discovered, and which, as far as I know, has not been thought of by others. It is simply this: any wire, during drawing, is put under mechanical strain and forced into a certain condition which will gradually work itself out. The resistance of wires of all kinds is greater immediately after drawing than it will be a little later, speaking, of course, with reference purely to mechanical changes and not to chemical ones. The wire is hardened by drawing and slowly anneals by contact with ordinary atmospheric temperatures. The same is true of glass as is well illustrated by the changes which take place in thermometers after they have first been made, unless the glass used has been kept in the tube form for a great length of time. Good thermometer makers consider it necessary to hold glass for several years before being used, if the thermometers are to be really standard. All this is not true, however, of quartz, and if it were possible to obtain quartz in the right shape or to get it into that shape and work it, we should be able to make thermometers almost absolutely correct and which would remain so. Now, let us again look at the system shown in *Fig. 5*. Here there are two sets of magnets, or, essentially, two magnets, with their poles in opposite directions. Let *Fig. 10* represent, diagramatically, these two magnets opposed to one another but not quite in the same plane. These two magnets are evidently equivalent to another magnet whose plane will be somewhere in the angle θ; let us suppose that it lies in the position indicated by the dotted line; this dotted line will, therefore, lie in the plane of the earth's meridian, assuming for convenience that no other field but that of the earth is operative. If, now, either of the magnets shifts its plane with reference to the other there will be a corresponding shifting of the resultant plane or the *system* will rotate until the *new* plane is in the earth's meridian. We will, therefore, have "drift" again but due to another cause. If we use aluminum or other metal wire the twist produced by the original drawing of the wire will slowly work itself out, perhaps taking months to do so, and we will always be more or less bothered by the consequent drift. Glass is equally bad as metal. Quartz, however, will not alter and is perfect

for this purpose. To prove our hypothesis we took aluminum wire upon which was a system, and in which there was but little drift, and gave the wire a slight twist; drift invariably followed and in the direction which theory would lead us to expect from the direction of twisting.

BOOK NOTICES.

Recent Progress in Electric Railways: being a summary of current periodical literature relating to electric railway construction, operation, systems, machinery, appliances, etc. Compiled by Carl Hering. New York: The W. J. Johnston Company, Limited, 167–176 *Times* Building. London: Whittaker & Co., Paternoster Square. 1892. Price, $1.

The publishers of *The Electrical World* contemplate the publication, from time to time, of a series of small volumes embracing a review or summary of current progress in the various branches of applied electricity, in which form the labor of search through the voluminous and inconvenient files of the electrical journals in which this information is now scattered will be avoided. Such a digest of information, prepared by skilful and discriminating compilers, should be of substantial benefit to active workers in the great electrical field, who would duly appreciate the saving of time and labor which it would mean to them. The volume above entitled is the first of the promised series, and forms a useful *résumé* of the subject of the electric railway, historical, statistical and technical. J. C.

Questions and Answers about Electricity. A first-book for students, etc. Authors: T. O'Connor Sloane, A.M., E.M., Ph.D., Caryl D. Haskins, M.I.E.E.; A. E. Watson; Edw. Trevert. Illustrated. New York: D. Van Nostrand Company. 1892.

This volume forms a 16mo of 100 pages, and is intended for the special service of the beginner. It is written in the style of a catechism, and covers the following chapter heads: "Theory of Electricity," "Theory of Magnetism," "Voltaic Batteries," "Dynamos and Motors," "Electric Lamps," "Electrical Appliances," "Electrical Measurement," and a "Glossary of Electrical Terms." The work of the authors appears to have been well done, and the book should prove extremely useful to the class for whom it is intended. J. C.

Encyclopédie Scientifique des Aide-Memoire, publiée sous la direction de M. H. Léauté, Membre de l'Institut. Paris: Gauthier-Villars et Fils et G. Masson. Libraires-Editeurs.

The scheme of the publishers in the announcement of this work contemplates the publication of about 300 volumes, in small 8vo form, to appear at the rate of about thirty or forty volumes per year. Each of these is

designed to be strictly practical in its character, and to represent at the hands of a distinguished expert the state of the art to which it relates. It will embrace the entire domain of the applied sciences—engineering, mechanics, electricity, physics, chemistry in one series, and agriculture, biology, medicine, surgery and hygiene in another. Each volume will be signed with the name of its author.

The following volumes of the mechanical, physical and chemical series have already appeared:

Dudebout, A. *Appareils d'essai des moteurs à vapeur; appareils d'asservissement.*
Witz, A. *Thermo-dynamique à l'usage des ingénieurs.*
Duquesnay. *Résistance des matériaux.*
Madamet, A. *Tiroirs et distributeurs de vapeur.*
Sauvage, Ed. *Les divers types de moteurs à vapeur.*
Madamet. *Détente variable de la vapeur. Dispositifs qui la produisent.*
Picou, R. V. *Distribution de l'électricité par usines centrales.*
Alheilig. *Recette, conservation et travail des bois. Outils et machines-outils employés dans ce travail.*
Gouilly, Al. *Transmission de la force motrice par air comprimé ou raréfié.*
Picou, R. V. *Distribution de l'électricité par installations isolées.*
Dwelshauvers-Dery. *Étude expérimentale calorimétrique de la machine à vapeur.*
Le Chatelier, H. *Le Grisou.*
Magnier de la Source, Dr. L. *Analyse des vins.*
Schloesing fils, Th. *Notions de chimie agricole.*
Lindel, Dr. L. *La bière.*

The contributions which have already appeared are the work of distinguished specialists, and fully justify the pledge of the publishers that the promised series should be encyclopædic and thoroughly reliable. A specially valuable feature of this encyclopædia, which will doubtless add greatly to its popularity, is the fact that the volumes will be sold separately, each comprising a single subject and being complete in itself. Another useful feature of the work is the addition at the close of each volume of a list of the principal works pertaining to the subject. W.

Experiments with Alternate Currents of High Potential and High Frequency. By Nikola Tesla. New York: The W. J. Johnston Company, Limited, 167 *Times* Building. 1892.

This volume is a reprint of the classic lecture delivered by its brilliant author, before the Institution of Electrical Engineers, London, and with which electricians of both hemispheres are familiar. The publication of the lecture in book-form by making it conveniently accessible to electrical engineers and students, should insure for the work a large demand. The work is prefaced by a portrait and biographical sketch of the author.

W.

The Galvanic Circuit Investigated Mathematically. By Dr. G. S. Ohm. Translated by William Francis, with a preface by the editor, Thomas D. Lockwood, M.I.E.E. New York: D. Van Nostrand Company, 23 Murray Street. 1891. Price, 50 cents.

The volume above-named constitutes No. 102 of the well-known Van Nostrand's Science Series, and is a translation of the classic researches of the illustrious scientist who first investigated and determined the quantitative relations of the galvanic circuit and formulated the same in the law to which his name is attached. Of the host of electricians who quote and use Ohm's law in daily lecture or practice, few have probably had the opportunity of reading the original investigations by which the law was established in 1827. This is due, so far as English and American students are concerned, in part to the circumstance that the only known translation of Ohm's original work by Francis (that of Taylor's *Scientific Memoirs*) is difficultly procurable. Mr. Lockwood has rendered a service to students of the subject by placing this masterly work at their service in such convenient form. W.

Die Elektrischen Verbrauchs-Messer. Von Etienne de Foder. Wien, Pest, Leipzig. A. Hartleben's Verlag. 1891.

The present volume on electric meters forms the forty-third of the well-known and popular "Elektro-technische Biblothek" series of this publishing house.

So far as we know this is the first attempt to collect and classify in book-form the data concerning the important subject of electric meters, and a glance through its pages will suffice to exhibit the fact that the author has displayed not only commendable zeal in his search through the literature to render his work as complete as possible, but also his thorough familiarity with the subject by his scientific arrangement and discussion of the merits of these instruments. It may be a surprise to some of our readers to know that the author describes more or less fully no less than ninety electric meters. The book is profusely illustrated and is an extremely useful addition to the literature of applied electricity. W.

Brick for Street Pavements. An account of tests made of bricks and paving blocks, with a brief discussion of street pavements and the method of constructing them. By M. D. Burke, C.E. Cincinnati: Robert Clarke & Co. 1892. 8vo. Paper. Price, 50 cents.

This work is in the form of a neatly-printed pamphlet with cover, being a reproduction of a report made by the author in his official capacity of engineer to the village of Avondale, O., on the tests made by him of various materials adapted for street paving. It embraces results of a series of tests of fifteen different varieties of clay products—bricks, paving blocks, etc. The samples submitted to examination were from as many different manufacturers, and may be regarded as representing fairly the average character of the paving bricks made in the United States.

The tests made by the author included the estimation of the following features: Chemical composition; absorption coëfficient for water; specific gravity; strength, transverse and crushing; and resistance to abrasion and impact. The author describes in full detail the methods pursued in making the tests, and presents his results in tabulated form. The report contains also an interesting discussion of the subject of street paving in general, and of the subject of the construction and maintainance of public works from the standpoint of the engineer.

The pamphlet should be useful not only to the engineer but also to the manufacturer of paving materials, containing as it does the results of a carefully conducted series of actual tests of materials in common use, and its utility is in nowise impaired by the author's intelligent comments on the general subject. W.

Ice-making Machines. The theory of the action of the various forms of cold-producing, or so-called ice machines. Translated from the French of M. Ledoux, *ingénieur des mines.* Revised and transformed to English units by J. E. Denton, D. S. Jacobus and A. Riesenberger. New York: D. Van Nostrand Company, 23 Murray Street. 1892. Price, 50 cents.

The essay of M. Ledoux, which originally appeared in the *Annales des Mines*, was translated and published in Van Nostrand's Science Series in 1879. It is a masterly discussion of the thermo-dynamic relations of the various fluids available for the production of cold, and the numerical results announced by him have been accepted by the leading manufacturers of refrigerating machinery.

Since the publication above stated, some of the constants have been re-determined experimentally, and the present translators have given the results of these newer determinations in an appendix and several new tables containing data of value to the student of the subject have been added. One feature which will make the work of Ledoux more available to American students has been introduced in the present edition, namely, the transformation of numerical values and results into English units. W.

The Railway Officials' Directory. A Guide for the Use of Railway Men and Dealers in Railway Supplies. Chicago: *Railway Age and Northwestern Railroader.* 1892.

This work, issued in pocket book form, and formerly known as *The Supply Men's Directory*, contains the addresses of Chairmen of Boards, Presidents, General Managers, General Superintendents, Purchasing Agents, Chief Engineers, Superintendents of Motive-power, Master Mechanics and Master Car-builders of all the railway companies of the United States, Canada and Mexico. The book is well indexed, making the task of reference easy. It must prove a useful publication for all whose professional and business pursuits demand an acquaintance with railway officials. The source from which it emanates should be sufficient guaranty for the accuracy of the compilation. W.

Franklin Institute.

[*Stated meeting held Wednesday, November 16, 1892.*]

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, November 16, 1892.

MR. J. M. WILSON, President, in the chair.

Present, 194 members and thirty visitors.

Additions to membership since last report, seventeen.

Mr. William E. Lockwood presented a paper, entitled "The Present Status of the Question of the Hammer-blow of a Locomotive's Driving Wheels; and as supplementary to the above, the Richards Balanced Locomotive, illustrated with working model under steam."

Mr. Lockwood's communications were fully illustrated with the aid of working models and lantern slides, and called forth considerable discussion.

Mr. Emil K. Winkler, manager of the Paul Von Janko Conservatory of Music, New York, exhibited a new key-board, invented by Mr. Paul Von Janko, and described its advantages over the old form of piano key-boards.

The communication of Professor Winkler was illustrated by a piano of Decker Brothers, of New York, furnished with the Von Janko key-board; a demonstration of the capabilities of the invention was given by Miss Newcomb, of New York, who executed a number of difficult compositions upon it with great success.

At the suggestion of the President, the invention was referred to the Committee on Science and the Arts for fuller investigation and suitable recognition of its merits.

Mr. Ives exhibited a number of color photographs with the heliochromoscope in connection with the originals for comparison of the truthfulness of the rendition of color.

Mr. Robert B. Haines, Jr., presented a brief communication, descriptive of a new automatic micrometer gauge, an invention for gauging the thickness of metal plates in rolling.

On account of the lateness of the hour, the Secretary's report was dispensed with, and the meeting adjourned.

WM. H. WAHL, *Secretary*.